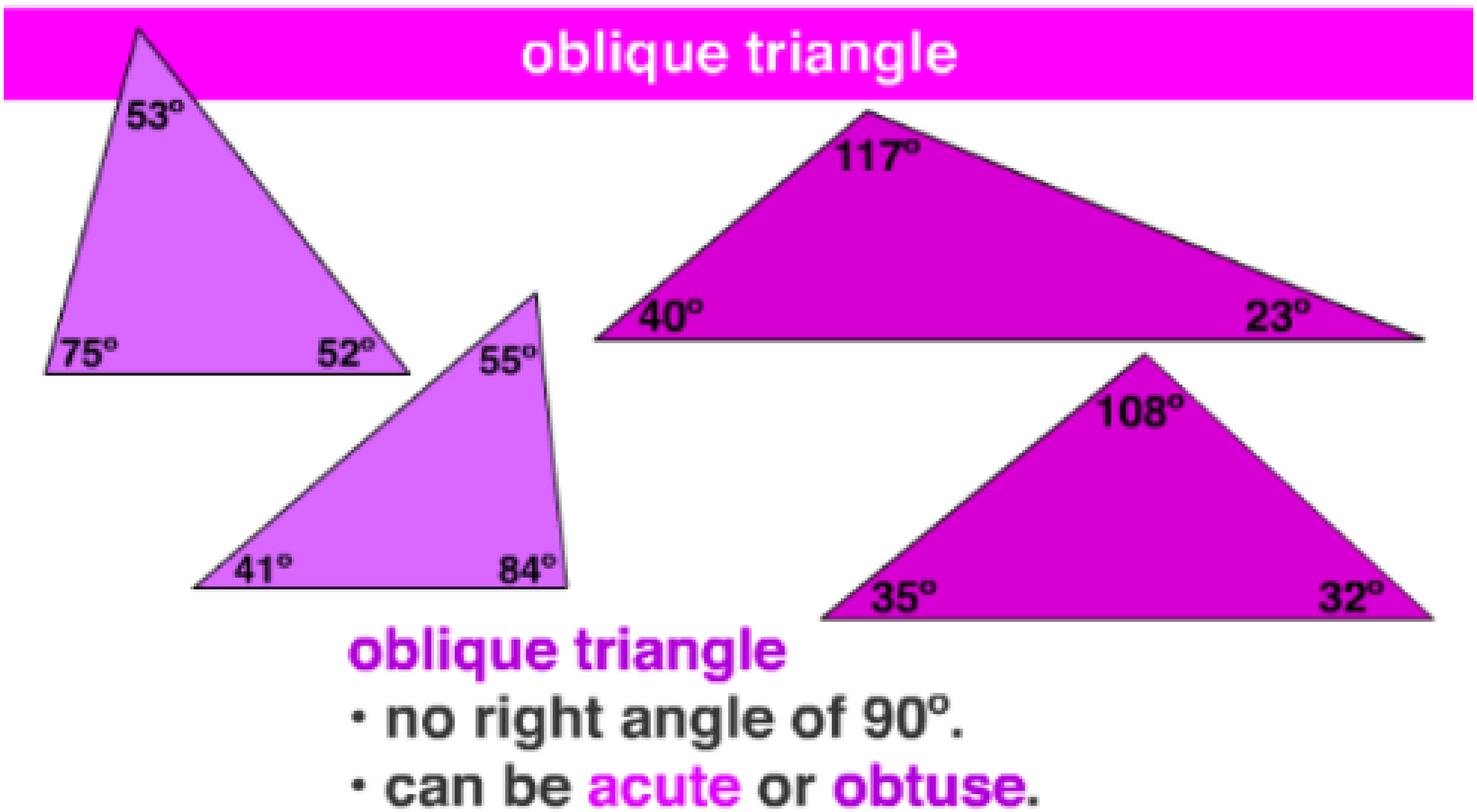
Chapter 3 / 4:

Oblique Triangle Trigonometry

Oblique triangles DO NOT have a right angle.



No right angle = no SOH CAH TOA

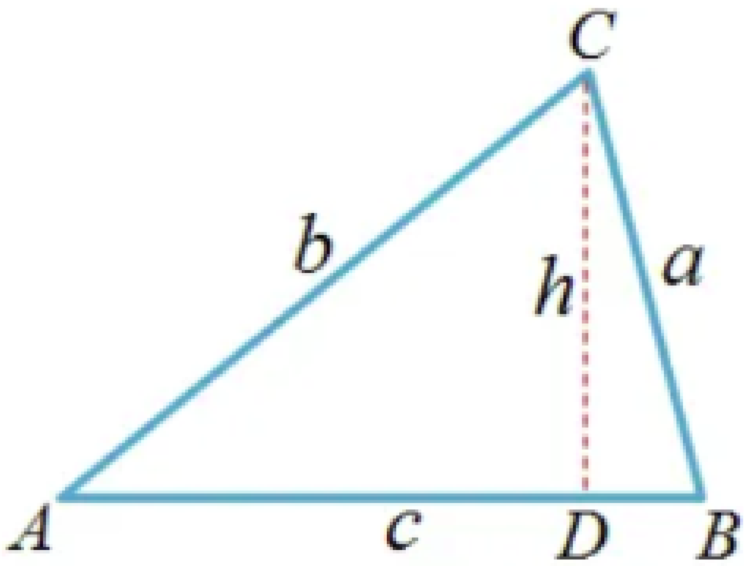
No right angle = no Pythagorean Thm

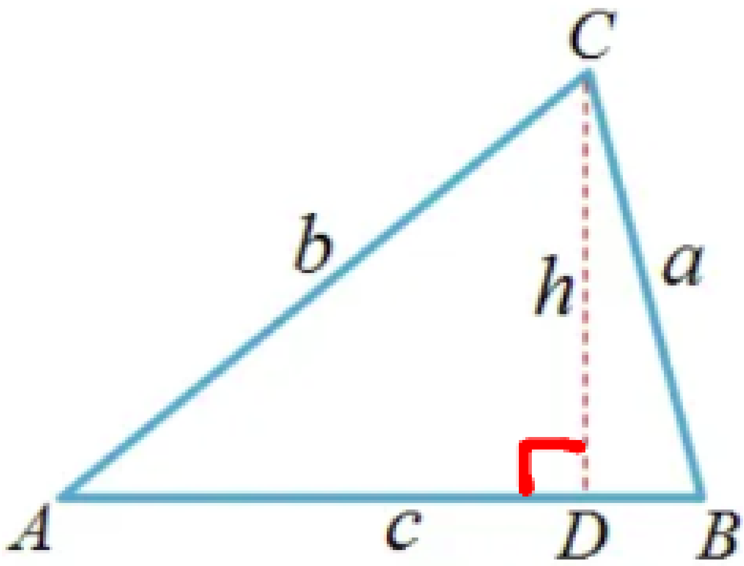
This unit will teach you two new rules for working with *oblique* triangles that will replace SOH CAH TOA and the Pythagorean Theorem.

The first of these two new rules is called:

The Sine Law

How did mathematicians come up with this new rule? They used known principles such as the sine ratio for right triangles.


C
b
h
a
A
c
D
B


C
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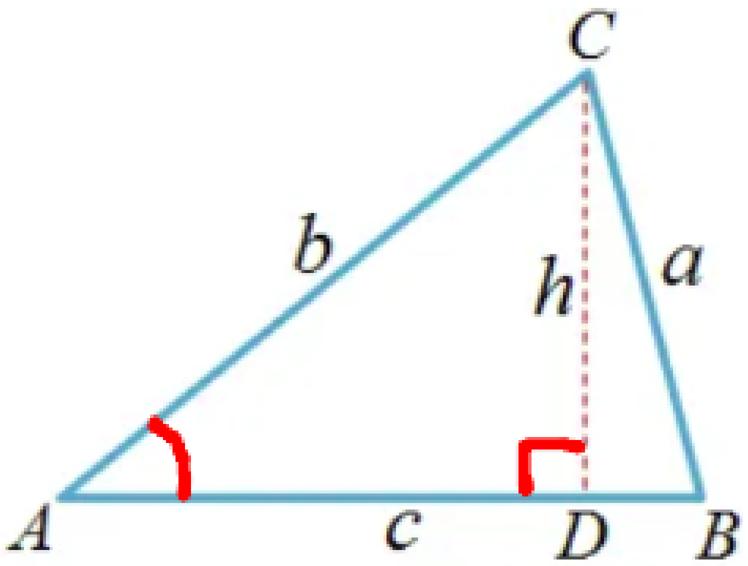
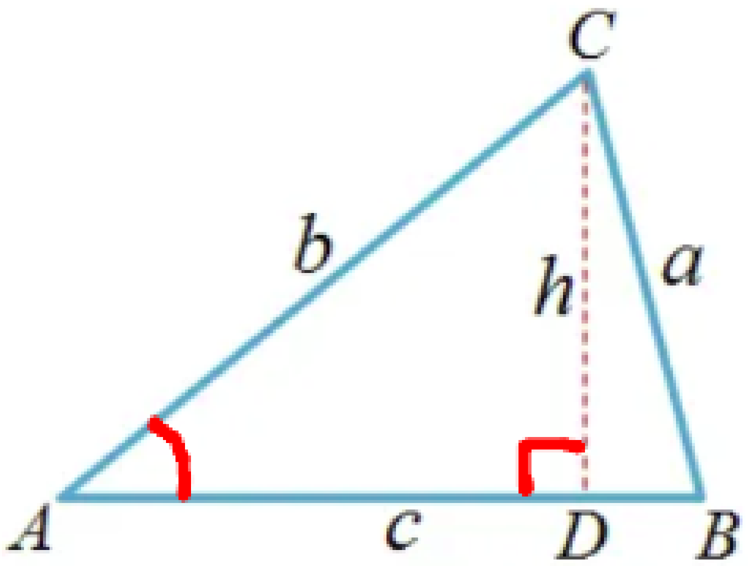
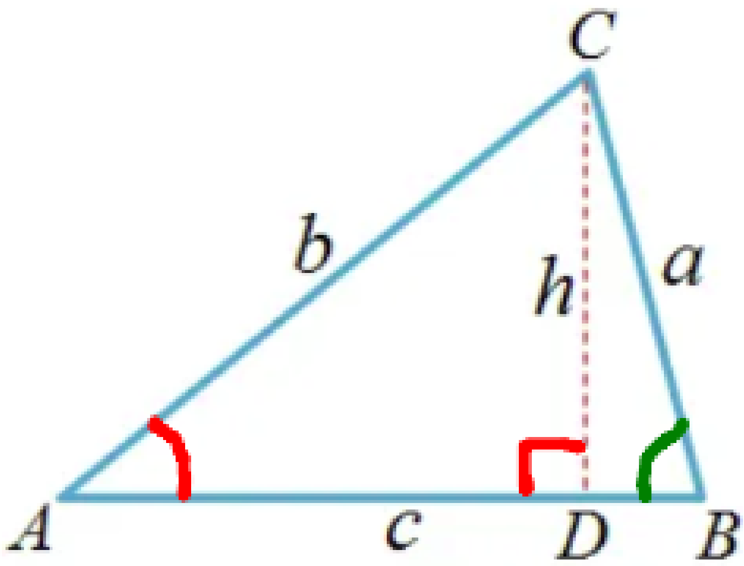


$$\sin A = \frac{h}{b}$$



$$b \sin A = \frac{h}{\cancel{b}} \cdot \cancel{b}$$

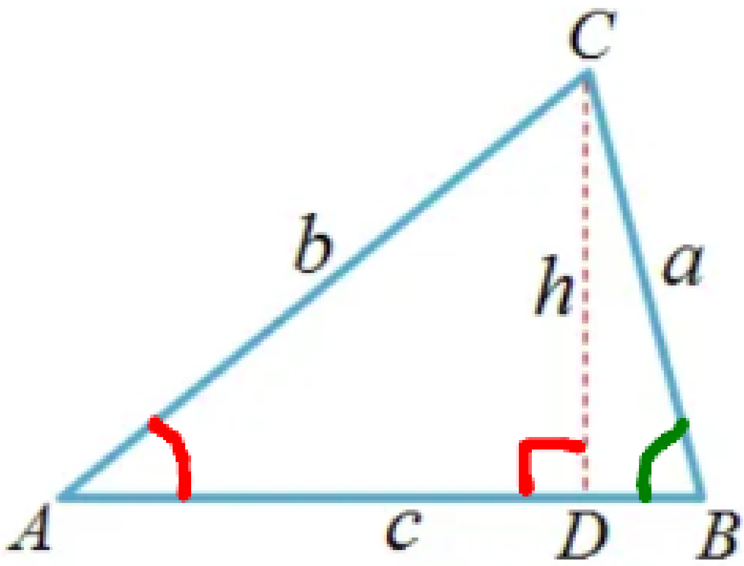
$$h = b \sin A$$



$$b\sin A = \frac{h}{\cancel{b}} \cdot \cancel{b}$$

$$\sin B = \frac{h}{a}$$

$$h = b\sin A$$

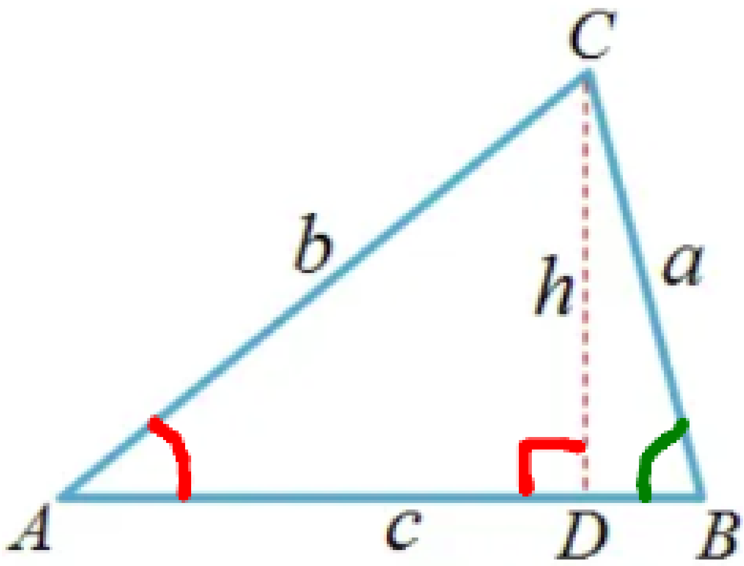


$$b \sin A = \frac{h}{\cancel{b}} \cdot \cancel{b}$$

$$a \sin B = \frac{h}{\cancel{a}} \cdot \cancel{a}$$

$$h = b \sin A$$

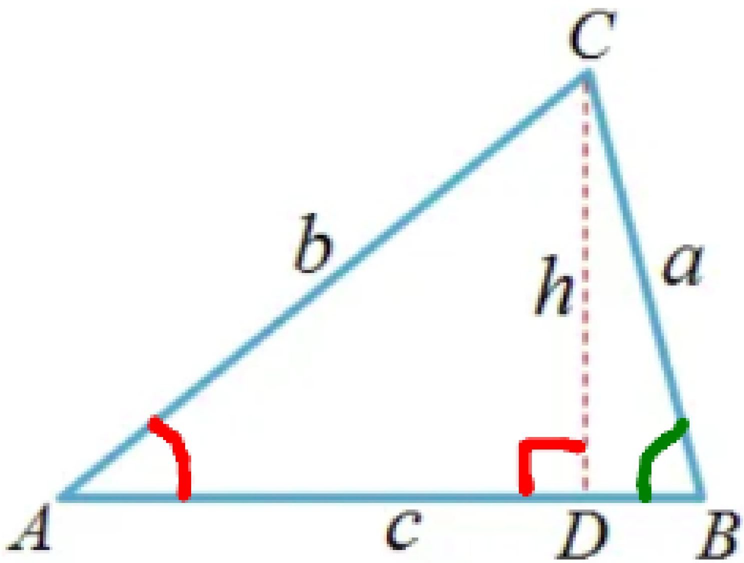
$$h = a \sin B$$



$$b \sin A = \frac{h}{\cancel{b}} \cdot \cancel{b} \qquad a \sin B = \frac{h}{\cancel{a}} \cdot \cancel{a}$$

$$h = b \sin A \qquad h = a \sin B$$

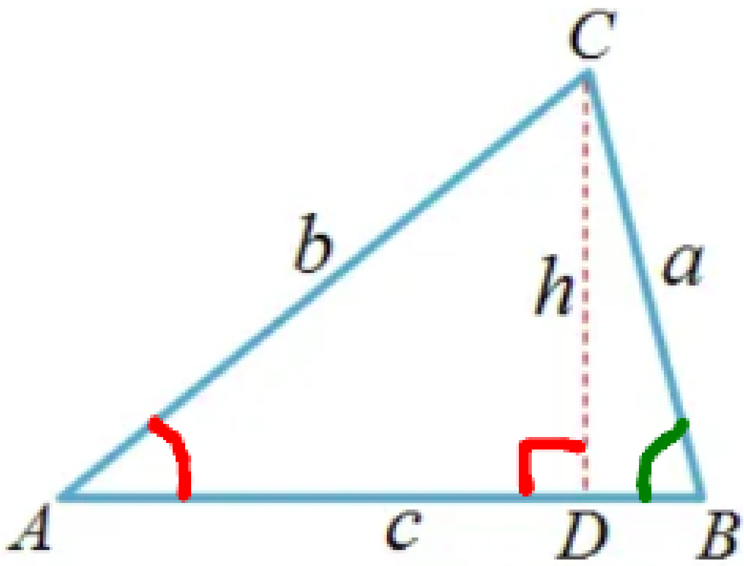
$$\therefore \quad b \sin A = a \sin B$$



$$b\sin A = \frac{h}{\cancel{b}}\cdot\cancel{b} \qquad a\sin B = \frac{h}{\cancel{a}}\cdot\cancel{a}$$

$$h = b\sin A \qquad h = a\sin B$$

$$\therefore \quad \frac{b\sin A}{ab} = \frac{a\sin B}{ab}$$



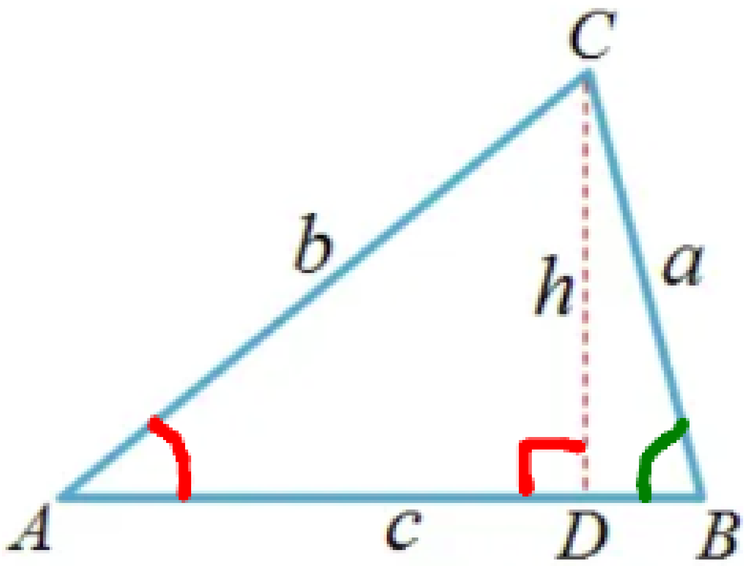
$$b\sin A = \frac{h}{\cancel{b}} \cdot \cancel{b}$$

$$a\sin B = \frac{h}{\cancel{a}} \cdot \cancel{a}$$

$$h = b\sin A$$

$$h = a\sin B$$

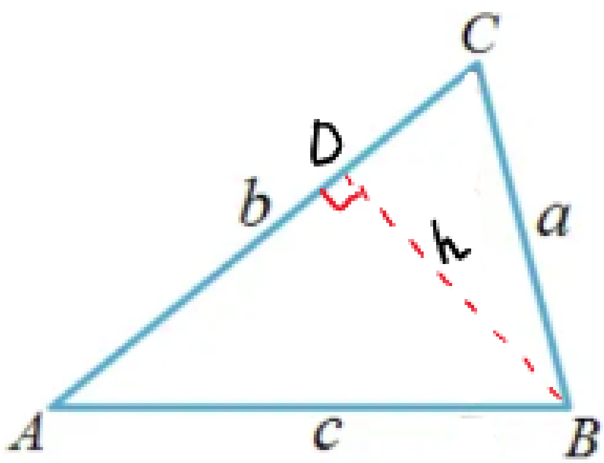
$$\therefore \quad \frac{\cancel{b}\sin A}{a\cancel{b}} = \frac{\cancel{a}\sin B}{\cancel{a}b}$$



$$b\sin A = \frac{h}{\cancel{b}} \cdot \cancel{b} \qquad a\sin B = \frac{h}{\cancel{a}} \cdot \cancel{a}$$

$$h = b\sin A \qquad h = a\sin B$$

$$\therefore \quad \frac{\cancel{b}\sin A}{a\cancel{b}} = \frac{\cancel{a}\sin B}{\cancel{a}b} \Rightarrow \frac{\sin A}{a} = \frac{\sin B}{b}$$


C
D
b
h
a
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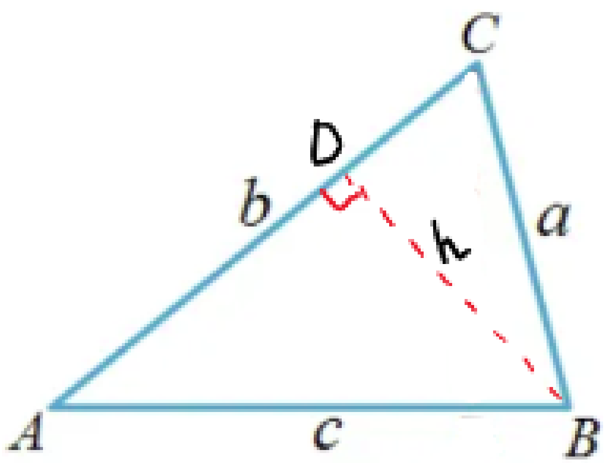
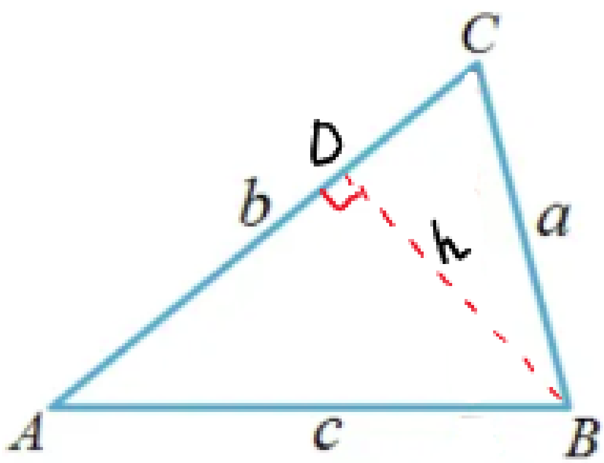
c
B

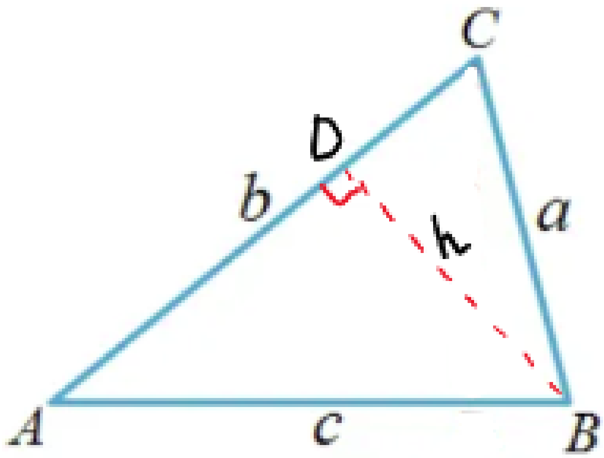


$$\sin A = \frac{h}{c}$$

$$\sin C = \frac{h}{a}$$



$$c \cdot \sin A = \frac{h}{\cancel{c}} \cdot \cancel{c} \qquad a \cdot \sin C = \frac{h}{\cancel{a}} \cdot \cancel{a}$$

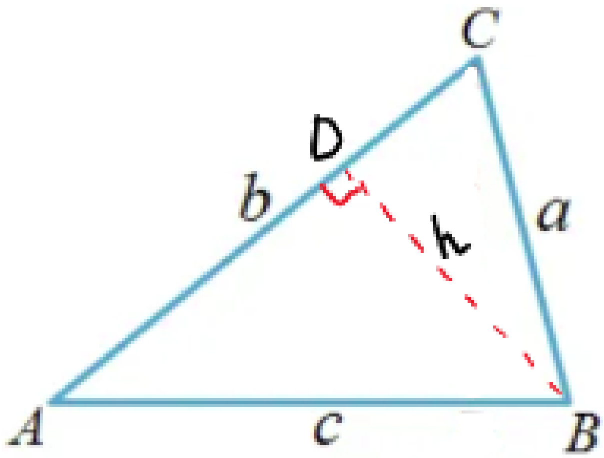


$c \cdot \sin A = \frac{h}{\cancel{c}} \cdot \cancel{c}$

$h = c\sin A$

$a \cdot \sin C = \frac{h}{\cancel{a}} \cdot \cancel{a}$

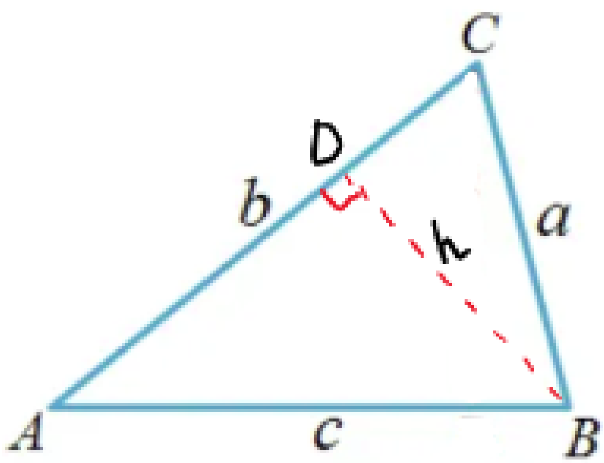
$h = a\sin C$



$$c \cdot \sin A = \frac{h}{\cancel{c}} \cdot \cancel{c} \qquad a \cdot \sin C = \frac{h}{\cancel{a}} \cdot \cancel{a}$$

$$h = c \sin A \qquad h = a \sin C$$

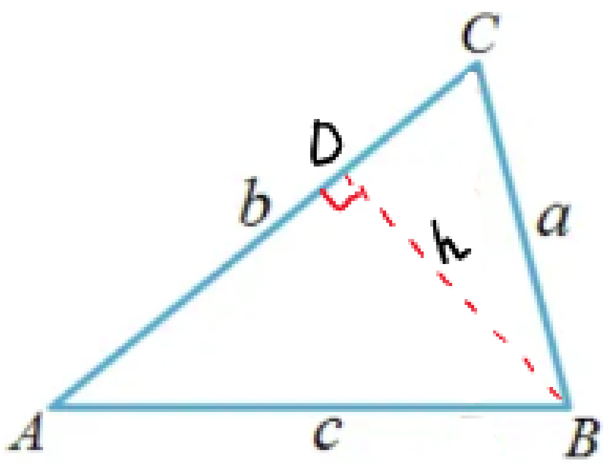
$$\therefore \quad c \sin A = a \sin C$$



$$c \cdot \sin A = \frac{h}{\cancel{c}} \cdot \cancel{c} \qquad a \cdot \sin C = \frac{h}{\cancel{a}} \cdot \cancel{a}$$

$$h = c\sin A \qquad h = a\sin C$$

$$\therefore \quad \frac{\cancel{c}\sin A}{a\cancel{c}} = \frac{\cancel{a}\sin C}{\cancel{a}c}$$



$$c \cdot \sin A = \frac{h}{\cancel{c}} \cdot \cancel{c} \qquad a \cdot \sin C = \frac{h}{\cancel{a}} \cdot \cancel{a}$$

$$h = c\sin A \qquad h = a\sin C$$

$$\therefore \quad \frac{\cancel{c}\sin A}{a\cancel{c}} = \frac{\cancel{a}\sin C}{\cancel{a}c} \Rightarrow \frac{\sin A}{a} = \frac{\sin C}{c}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

and

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

$$\therefore \quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

This formula is the Sine Law

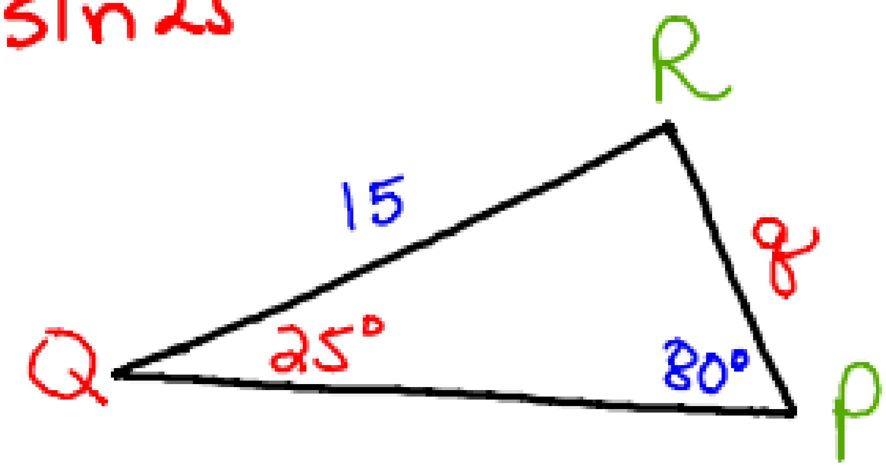
When we use this rule we will be solving a proportion to find a missing side length or a missing angle measure.

Solve for the unknown side length to one decimal place:

$$\frac{q}{\sin 25°} = \frac{15}{\sin 80°}$$

Solve for the unknown side length to one decimal place:

$$\cancel{\sin 25^\circ}\,\frac{q}{\cancel{\sin 25^\circ}} = \frac{15}{\sin 80^\circ}\cdot \sin 25^\circ$$



$$q = \frac{15\sin 25^\circ}{\sin 80^\circ}$$

$$q = 6.43706\ldots \Rightarrow \boxed{q = 6.4}$$

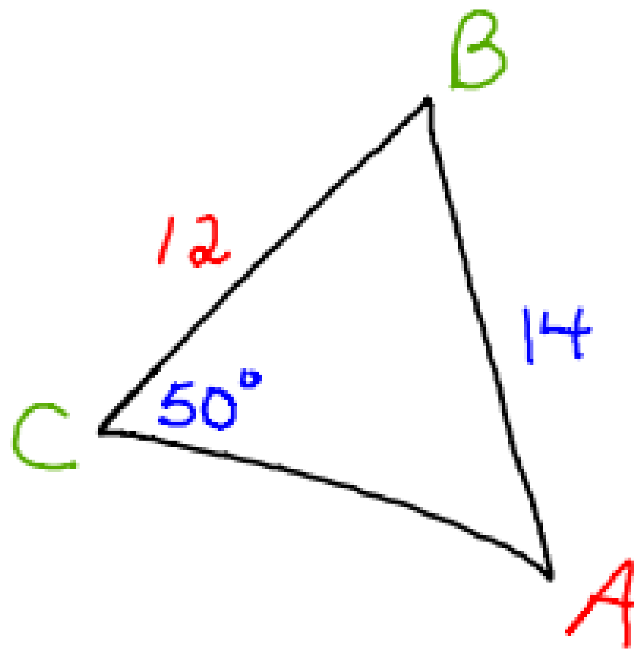
Solve for the unknown angle measure to the nearest degree:

$$\frac{12}{\sin A} = \frac{14}{\sin 50°}$$

Solve for the unknown angle measure to the nearest degree:

$$\frac{12}{\sin A} = \frac{14}{\sin 50^\circ}$$

"flip"



$$\cancel{12}\cdot\frac{\sin A}{\cancel{12}} = 12\cdot\frac{\sin 50^\circ}{14}$$

$\sin^{-1}$ $\sin^{-1}$

$$\sin A = \frac{12\sin 50^\circ}{14} = 0.6566095\ldots$$

$$\angle A = 41.041805\ldots \rightarrow \boxed{\angle A = 41^\circ}$$

Check your understanding:

Handout:

#1(b)(c)(d)(e)(f), 2(b)(d)(e)

I highly encourage you to do the sketches, and not just solve for the side or angle.