Chapter 5:

Introduction to Polynomials

Unit Overview:

You will learn:

1) *Terminology* related to polynomials:
   - types of polynomials
   - degree of a polynomial
   - like terms
2) To add and subtract polynomials by *combining like terms*.
3) Apply polynomials to perimeters of simple geometric shapes.
4) Use polynomials to *model* real-world situations.

Section 5.1: The Language of Mathematics

Learning Targets:

1. Use mathematical terminology to describe polynomials with respect to:
    - their number of terms
    - their variables
    - their constant
    - their degree
2. Create a polynomial expression to match a given description.

Key Terms:

Algebra - a branch of math that uses symbols to represent unknown numbers. These symbols are called variables.

Variable - a letter or symbol that represents an *unknown* number.

Constant - an expression that is simply a number - it is a *known* value in an expression or equation.

Coefficient - a number that multiplies a variable or variables.

Algebraic Expressions

Mathematical expressions using a combination of numbers, variables and mathematical operations.

Describe the following expression in terms of its variable(s), coefficient(s), and constant:

$x + 5$

the variable is x

the coefficient on x is 1

the constant is 5

Algebraic Expressions

Mathematical expressions using a combination of numbers, variables and mathematical operations.

Describe the following expression in terms of its variable(s), coefficient(s), and constant:

$$x - y - 6$$

$$x + (-y) + (-6)$$

the variables are x and y

the coefficient on x is 1

the coefficient on y is -1

the constant is -6

Algebraic Expressions

Mathematical expressions using a combination of numbers, variables and mathematical operations.

Describe the following expression in terms of its variable(s), coefficient(s), and constant:

$a^2 + 15$

the variable is a

the coefficient on a^2 is 1

the constant is 15

Algebraic Expressions

Mathematical expressions using a combination of numbers, variables and mathematical operations.

Describe the following expression in terms of its variable(s), coefficient(s), and constant:

$$\underbrace{4x}_{4x+0}$$

the variable is x

the coefficient is 4

the constant is 0

Algebraic Expressions

Mathematical expressions using a combination of numbers, variables and mathematical operations.

Describe the following expression in terms of its variable(s), coefficient(s), and constant:

$$7 - 2y$$

$$7 + (-2y)$$

the variable is y

the coefficient on y is -2

the constant is 7

Algebraic Expressions

Mathematical expressions using a combination of numbers, variables and mathematical operations.

Describe the following expression in terms of its variable(s), coefficient(s), and constant:

$$\underbrace{a^2 + 2b}$$

$a^2 + 2b + 0$

the variables are a and b

the coefficient on a^2 is 1

the coefficient on b is 2

the constant is 0

Algebraic Expressions

Mathematical expressions using a combination of numbers, variables and mathematical operations.

You Try:

Describe the following expression in terms of its variable(s), coefficient(s), and constant:

$$-5x - 10$$

Algebraic Expressions

Mathematical expressions using a combination of numbers, variables and mathematical operations.

You Try:

Describe the following expression in terms of its variable(s), coefficient(s), and constant:

$-5x - 10$

the variable is "x"
the coefficient on x is "-5"
the constant is "-10"

Term: an expression formed by the product of a coefficient and a variable or multiple variables. In algebraic expressions, terms are linked together by addition and subtraction.

An algebraic expression could have only one term, or many terms.

Single terms (examples)

5 (every constant is one term) $-6, \frac{1}{2}, 0.2$, etc.

n (a variable by itself is one term) x, y, a, b, etc.

7x (a variable with a coefficient is one term) $-3a, \frac{1}{3}k$

This expression means "7 times x"

-3ab (a coefficient with multiple variables is one term) $5xy$

This expression means "-3 times a times b"

$4m^2$ (a variable power with a coefficient is one term) $-3m^2, 50x^2$

This expression means "4 times m^2"

Two terms:

$5 + x$ (two terms linked together with addition) $y + x$

$3x^2 - 2$ (two terms linked together with subtraction) $4x - 3$

$7xy + z^2$ (two terms linked together with addition) $5k^2 + 4k$

Three terms:

$1 - x + y$ (three terms linked together with subtraction & addition)

$2x^2 + 3x - 5$ (three terms linked together with addition & subtraction)

$a + b + c$ (three terms linked together with addition)

Example:

How many terms are in each of these four algebraic expressions:

$4xy + 3$

2 terms

$7a^2 - 2ab + b^2$

3 terms

$5x^2 + y^2 + z^2 - x - 6$

5 terms

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1 term

The algebraic expressions we have been describing have another name:

POLYNOMIALS

A polynomial is an algebraic expression made up of any number of terms that are connected by the operations of addition and/or subtraction.

POLYNOMIALS

Polynomials are classified in a number of different ways. One way is by how many terms they have:

1 term = monomial

2 terms = binomial

3 terms = trinomial

POLYNOMIALS

Polynomials with more than 3 terms don't have particular names.

We simply call them "polynomials" and refer to the number of terms they have.

EX: $3x^2 - 4y + 2x - 7$ is a "4-term polynomial"

Example:

Classify the polynomials by their number of terms:

$7x - 2$ binomial

$-3xy$ monomial

$5x^2 + 8x - 2y + 6$ 4-term polynomial

$8m^2 + 4m - 9$ trinomial

Degree:

Polynomials can also be classified according to their "degree".

In order to determine the degree of a polynomial, we first need to know how to find the degree of a single term.

Degree of a term:

Every term of a polynomial has its own degree:

- add up all of the exponents on all of the variables in the term

Constant terms: degree = 0

5, -3, 9, 0.25, etc.

Single variable terms: degree = 1

x, 3y, -a, 4n, etc.

Degree of a term:

Power terms: degree = whatever the exponent is

m^2 degree = 2
$3x^2$ degree = 2
$-y^3$ degree = 3

Terms containing multiple variables:
degree = add up all of the exponents

$5ab$ degree = 2
$-2x^2y$ degree = 3
abc degree = 3

Degree of a polynomial:

The degree of a polynomial is equal to the highest degree of its terms.

Find the degree of each term - pick the highest value. That is the degree of the polynomial.

EX: What is the degree of $3x - 4xy + 7$?

degree 1 ↙ $3x$; degree 0 ↙ 7

$4xy$: degree = 2 is the highest degree of all the terms

∴ the degree of this polynomial is 2.

Example:

Classify each polynomial according to their number of terms and state the degree of the polynomial:

$4x^2 + 3$ degree 2 binomial

$7a^2 - 8ab + b^2$ degree 2 trinomial

Example:

Classify each according to their number of terms and state the degree of the polynomial:

-10

degree 0 monomial

$5z - 3 + 2w$

degree 1 trinomial

Writing polynomials:

Just like there are proper ways to write things in English, there are proper ways to write things in mathematical notation.

When working with polynomials, focus on:

(1) always put coefficients in front of variables, not behind (3a not a3)

(2) terms with multiple variables should have the variables listed alphabetically (5ab not 5ba)

(3) polynomials with multiple terms should be written so that the terms are written in order of *descending degree*. If multiple terms have the same degree, it doesn't matter which order they are written as long as lower degree terms are written later.

Example:

Write a polynomial to match the description:

(1) binomial of degree 1 with two different variables

$5a + 2b$

(2) degree 2 monomial with a negative coefficient

$-ab$

(3) degree 0

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Check your understanding questions:

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#5 - 10, 15 ***(model with an expression, not a diagram),***
#19, 28, 30