

Pre-calculus 20

Final Review

An arithmetic sequence begins with -5 and increases by 7 each time.

a) What is the 82nd term of the sequence?

$$t_1 = -5$$

$$n = 82$$

$$d = 7$$

$$t_n = t_1 + (n-1)d$$

$$t_{82} = -5 + (82-1)(7)$$

$$t_{82} = -5 + 81(7)$$

$$t_{82} = 562$$

b) What is the sum of the first 22 terms of this sequence?

$$t_1 = -5$$

$$d = 7$$

$$n = 22$$

$$S_n = \frac{n}{2} [2t_1 + (n-1)d]$$

$$S_n = \frac{22}{2} [2(-5) + (22-1)(7)]$$

$$S_{22} = 11 (-10 + 147)$$

$$S_{22} = 11 (137)$$

$$S_{22} = 1507$$

Find the 7th term of the sequence:

27, 18, 12, ...

$$\frac{18}{27} = \frac{2}{3}$$

$$\frac{12}{18} = \frac{2}{3}$$

∴ geometric

$$r = \frac{2}{3}$$

$$t_1 = 27$$

$$n = 7$$

$$t_n = t_1 r^{n-1}$$

$$t_7 = 27 \left(\frac{2}{3}\right)^{7-1}$$

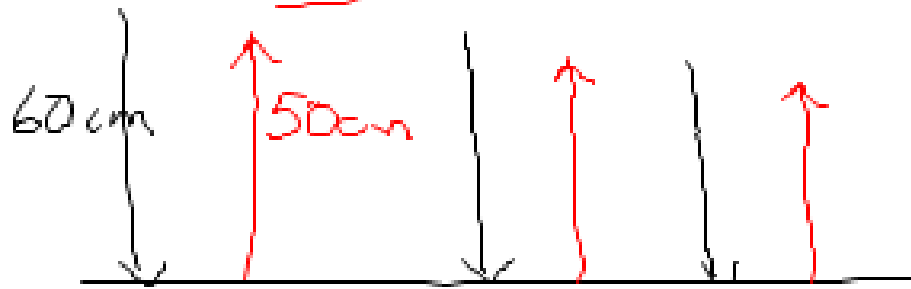
$$t_7 = 27 \left(\frac{2}{3}\right)^6$$

$$t_7 = \cancel{27} \left(\frac{64}{\cancel{729}}\right)$$

$$t_7 = \frac{64}{27}$$

A ball is dropped from height of 60 cm. The ball bounces back up to a height that is $\frac{5}{6}$ of the previous bounce.

a) How high does the ball bounce on its 4th bounce? (term)



$$\dots \quad t_n = t_1 r^{n-1}$$

$$t_1 = 50$$

$$t_4 = 50 \left(\frac{5}{6} \right)^3$$

$$t_4 = 50 \left(\frac{5}{6} \right)^{4-1}$$

$$r = \frac{5}{6}$$

$$t_4 = \frac{3125}{108}$$

$$t_4 = 50 \left(\frac{5}{6} \right)^3$$

$$n = 4$$

$$= 28.9 \text{ cm}$$

b) If the ball bounces indefinitely, how far will it travel altogether?

S_{∞} for bounces + S_{∞} for drops

$$S_{\infty} = \frac{t_1}{1-r}$$

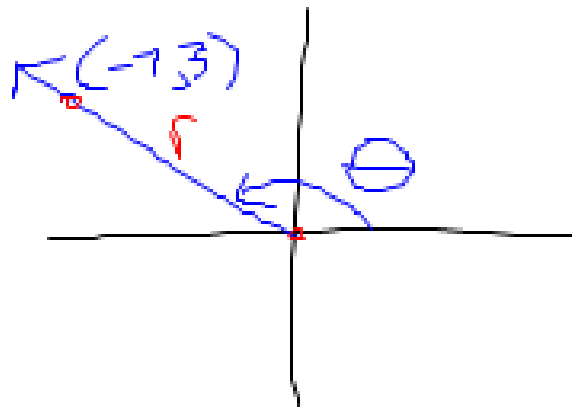
$$\frac{50}{1 - \frac{5}{6}} + \frac{60}{1 - \frac{5}{6}}$$

$$\begin{aligned} \frac{50}{\frac{1}{6}} + \frac{60}{\frac{1}{6}} &= 50(6) + 60(6) \\ &= 300 + 360 = \underline{660 \text{ cm}} \end{aligned}$$

SYR OXR TYX

The point $(-7, 3)$ lies on the terminal arm of an angle θ in standard position. Determine the exact values for the three primary trig ratios for angle θ .

sin cos tan



$$r = \sqrt{x^2 + y^2}$$

$$r = \sqrt{(-7)^2 + 3^2}$$

$$r = \sqrt{49 + 9}$$

$$r = \sqrt{58}$$

$$x = -7$$

$$y = 3$$

$$r = \sqrt{58}$$

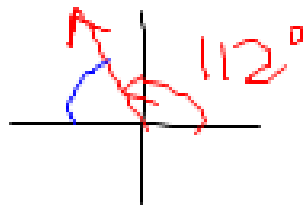
$$\sin \theta = \frac{3}{\sqrt{58}}$$

$$\cos \theta = \frac{-7}{\sqrt{58}}$$

$$\tan \theta = -\frac{3}{7}$$

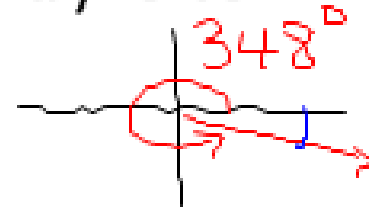
Calculate the reference angle for each of the following angles in standard position:

a) 112°



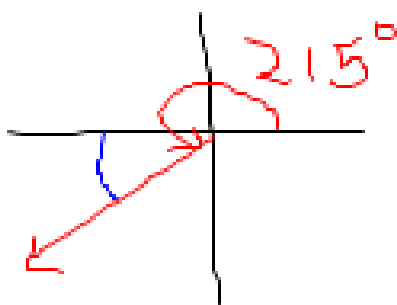
$$180^\circ - 112^\circ = 68^\circ$$

b) 348°



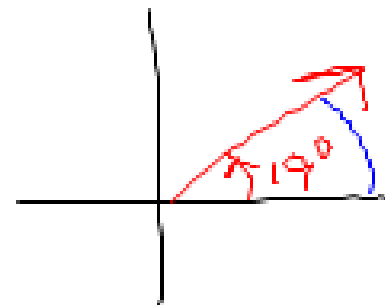
$$360^\circ - 348^\circ = 12^\circ$$

c) 215°



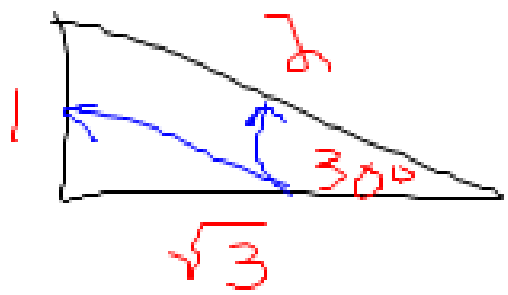
$$215^\circ - 180^\circ = 35^\circ$$

d) 19°



$$19^\circ$$

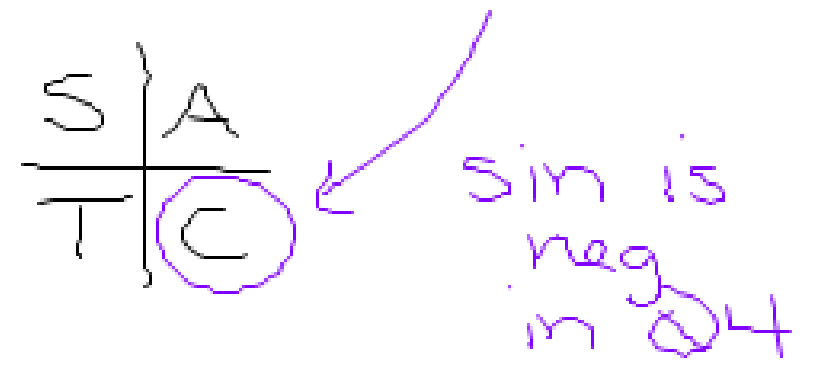
Calculate the exact value of $\sin 330^\circ$



$$\Theta_R = 360^\circ - 330^\circ = 30^\circ$$

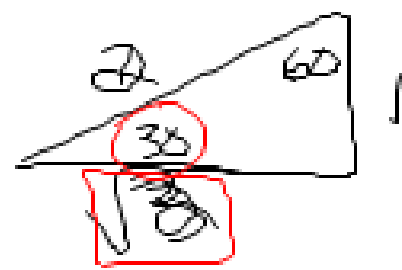
$$\sin 30^\circ = \frac{1}{2}$$

$330^\circ \Rightarrow Q4$



$$\therefore \sin 330^\circ = -\frac{1}{2}$$

positive
 If $\cos \theta = \frac{\sqrt{3}}{2}$, what are the two measures of $\angle \theta$?



| | |
|---|---|
| S | A |
| T | C |

$$\theta_1 = 30^\circ \quad (\text{Q1})$$

$$\theta_2 = 330^\circ \quad (\text{Q4})$$

$$\therefore \theta_R = 30^\circ$$

negative
 If $\tan \theta = -2.4567$, what are the two measures of $\angle \theta$?

$$\theta_R = \tan^{-1}(2.4567) = 68^\circ$$

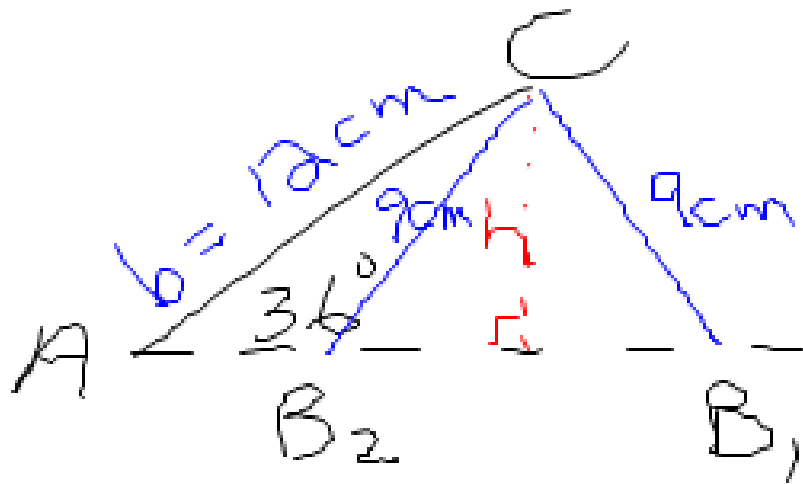
| | |
|---|---|
| S | A |
| T | C |

$$\theta_1 = 112^\circ \quad (\text{Q2})$$

$$\theta_2 = 292^\circ \quad (\text{Q4})$$

Solve the SSA ambiguous case:

$\triangle ABC$, $\angle A = 36^\circ$, $a = 9 \text{ cm}$, $b = 12 \text{ cm}$



$$a < b$$

$$h = 12 \sin 36^\circ = 7$$

$$a > h$$

$\therefore 2 \triangle's$

$$\frac{\sin B_1}{12} = \frac{\sin 36^\circ}{9}$$

$$\angle B_1 = \sin^{-1} \left(\frac{12 \sin 36^\circ}{9} \right)$$

$$\angle B_1 = 52^\circ$$

$$\angle B_2 = 180^\circ - 52^\circ = 128^\circ$$

$$\angle C = 180^\circ - 36^\circ - 52^\circ$$

$$\angle C = 92^\circ$$

$$\frac{c}{\sin 92^\circ} = \frac{9}{\sin 36^\circ}$$

$$c = \frac{9 \sin 92^\circ}{\sin 36^\circ}$$

$$c = 15.3 \text{ cm}$$

$$\angle C = 180^\circ - 36^\circ - 128^\circ$$

$$\angle C = 16^\circ$$

$$\frac{c}{\sin 16^\circ} = \frac{9}{\sin 36^\circ}$$

$$c = \frac{9 \sin 16^\circ}{\sin 36^\circ}$$

$$c = 4.2 \text{ cm}$$