**Pre-Calculus 30**

## 8.1 – Understanding Logarithms

In section 1.4, you were introduced to the inverse of a function. The inverse of a function is created by interchanging the abscissa and the ordinate of the ordered pair ( $x$ *and* $y$ ). When creating the inverse from a function that is already graphed for us, we see that the points are a reflection in the line $\boxed{y = x}$

What is the inverse of the exponential function $y = 2^x$ ? We will first explore this answer graphically.

- Will the inverse of this exponential function also be a function? Explain.

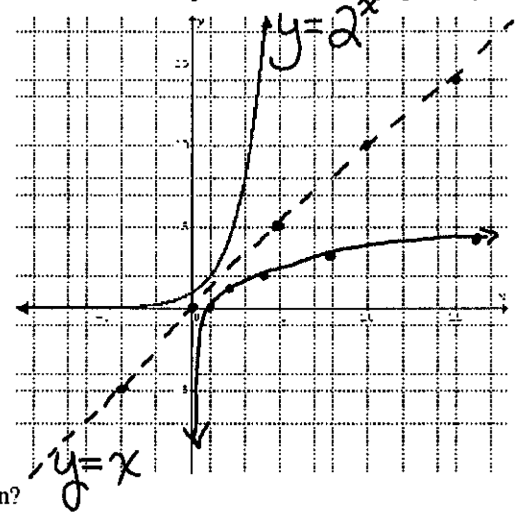
  Yes. Because $y=2^x$ passes the HLT.

- Add the line $y = x$ to the graph.
- Identify key points for the function $y = 2^x$.

| x | y |
|---|---|
| 0 | 1 |
| 1 | 2 |
| 2 | 4 |

| x | y |
|---|---|
| 1 | 0 |
| 2 | 1 |
| 4 | 2 |

- Interchange the abscissa and the ordinates to create the inverse. Plot the points on the graph.



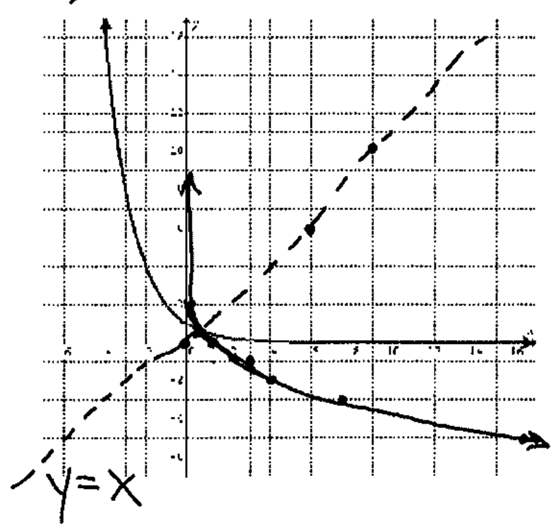

- What are the characteristics of the inverse function?

  x-int at (1,0)
  no y-int → vertical asymptote at the y-axis (x=0)
  increasing → Q4 → Q1
  D: $\{x>0, x\in\mathbb{R}\}$ R: $\{y\in\mathbb{R}\}$

Repeat the process for the graph of $y = \left(\frac{1}{2}\right)^x$.

| x | y |
|---|---|
| 0 | 1 |
| 1 | 1/2 |
| 2 | 1/4 |
| -1 | 2 |
| -2 | 4 |
| -3 | 8 |
| -4 | 16 |

| x | y |
|---|---|
| 1 | 0 |
| 1/2 | 1 |
| 1/4 | 2 |
| 2 | -1 |
| 4 | -2 |
| 8 | -3 |
| 16 | -4 |



Now we will use algebra to create the equation of this inverse function.

- Start with the original function $\boxed{y = 2^x.}$
- Create the inverse by switching the position of the variables $x$ and $y$. → $x = 2^y$
- Now some operation must be performed to isolate y.

?

As there is no operation that exists to isolate y, logarithms were created. The inverse of the function $y = c^x$ is $x = c^y$. In logarithmic form, the inverse is written $\boxed{y = \log_c x.}$ $y = \log_2 x$

- It is said "$y$ is the logarithm of $x$ in base 2".
- As a logarithmic function is the inverse of an exponential function, the same restrictions apply for c. The value of base c can be any positive real number except 1.
- Using function notation, if $f(x) = c^x$ then $f^{-1}(x) = \log_c x$.

Exponential equations can be rewritten in logarithmic form and logarithmic equations can be rewritten in exponential form.

| **Exponential Form** | **Logarithmic Form** |
|---|---|
| $7^4 = 2401$ | $\log_7 2401 = 4$ |
| $5^3 = 125$ | $\log_5 125 = 3$ |
| $5^7 = 78125$ | $\log_5 78\,125 = 7$ |
| $\left(\frac{1}{2}\right)^{-4} = 16$ | $\log_{\frac{1}{2}} 16 = -4$ |
| $p^q = r$ | $\log_p r = q$ |
| $j^m = h$ | $\log_j h = m$ |

※ Notice that a logarithm is the exponent.

As our number system is based on powers of ten, logarithms with base 10 are called ***common logarithms*** and we do not write 10 in the position of the base.

- $\log_{10} 10\,000 = 4$ is written as $\log 10\,000 = 4$

**Useful Observations:**

- $\log_c 1 = 0$ since in exponential form $c^0 = 1$.
- $\log_c c = 1$ since in exponential form $c^1 = c$
- $\log_c c^x = x$ since in exponential form $c^X = c^X$ (inverse property)
- $c^{\log_c x} = x, x > 0$, since in logarithmic form $\log_c x = \log_c x$ (inverse property)

***Examples:***

1. Evaluate the following logarithmic expressions.

a) $\log_2 32 = x$

$2^x = 32$

$2^x = 2^5$

$x = 5$

b) $\log 1\,000\,000 = x$

$10^x = 1000000$

$10^x = 10^6$

$x = 6$

c) $\log_2\left(\frac{1}{8}\right) = x$

$2^x = \frac{1}{8}$

$2^x = 2^{-3}$

$x = -3$

d) $\log_3 9\sqrt{3} = x$

$3^x = 9\sqrt{3}$

$3^x = 3^2 \cdot 3^{1/2}$

$3^x = 3^{5/2}$

$x = 5/2$

e) $\log_9 \sqrt[5]{81} = x$

$9^x = \sqrt[5]{81}$

$9^x = \sqrt[5]{9^2}$

$9^x = (9^2)^{1/5} = 9^{2/5}$

$x = 2/5$

2. Determine the value of $x$.

a) $\log_4 x = -2$

$4^{-2} = x$

$\frac{1}{4^2} = x$

$\frac{1}{16} = x$

b) $\log_{16} x = -\frac{1}{4}$

$16^{-1/4} = x$

$\left(\frac{1}{16}\right)^{1/4} = x$

$\sqrt[4]{1/16} = x$

$\frac{1}{2} = x$

c) $\log_x 9 = \frac{2}{3}$

$x^{2/3} = 9$

$\left(\sqrt[3]{x^2}\right)^3 = (9)^3$

$\sqrt{x^2} = \sqrt{729}$

$x = \pm 27$

$x = 27$ (can't have a negative base)

***Application:***

Read the example on page 378.

3. The largest measured earthquake struck Chile in 1960. It measured 9.5 on the Richter scale. How many times as great was the seismic shaking of the Chilean earthquake than the 1949 Haida Gwaii earthquake, which measure 8.1 on the Richter scale?

$M = \log \frac{A}{A_0}$

$\frac{10^{9.5}}{10^{8.1}} = 10^{1.4}$

$\approx 25$ times stronger

Assign. p. 380-381

#1(b), 2, 3, 4, 12, 19