# *Lesson #4: Zero and Division (Section 1.3)*

## *Learning Targets:*

i) Meaningful division

ii) Implications of zero in the numerator and not in the denominator of an expression

iii) Implications of zero in the denominator and not in the numerator of an expression

iv) Implications of zero in both the numerator and denominator

v) x-intercept, vertical asymptote, or POD (hole)?

vi) Zero, undefined, or indeterminate?

# Meaningful Division

For division to be defined and have meaning, a given quotient must have only one result that checks. Thus $\frac{100}{4} = 25$, has meaning because $4 \times 25 = 100$.

# Zero in the Numerator But Not in the Denominator

$$\frac{0}{8} = 0$$

This has meaning because 0x8=0

$$\frac{n}{d} = 0 \text{ if } n = 0 \text{ and } d \neq 0.$$

**Application:** Find the x-intercepts of:

(a.k.a. zeroes of the function)

$$y = \frac{x^2 - 4}{x^2 + 1}$$

$= (x+2)(x-2)$

$\Rightarrow$ never equals zero

→ set the numerator to zero and Solve for x

$(x+2)(x-2) = 0$

$x = -2 \quad x = 2$

# Zero in the Denominator But Not in the Numerator

$$\frac{500}{0} = m$$

Is it possible to have 0xm=500?

$\frac{n}{d}$ is undefined if $d = 0$ and $n \neq 0$.

**Application:** find the equations of the vertical asymptotes

$$y = \frac{2}{x-3}$$

→ vertical asymptotes occur when the denom is zero

$x - 3 = 0$

$x = 3$ equation of the vertical asymptote

# Zero in Both the Numerator and the Denominator

$$\frac{0}{0} = k$$

Answer is not unique since 0 x anything is 0.

$$\frac{0}{0} \text{ is indeterminate.}$$

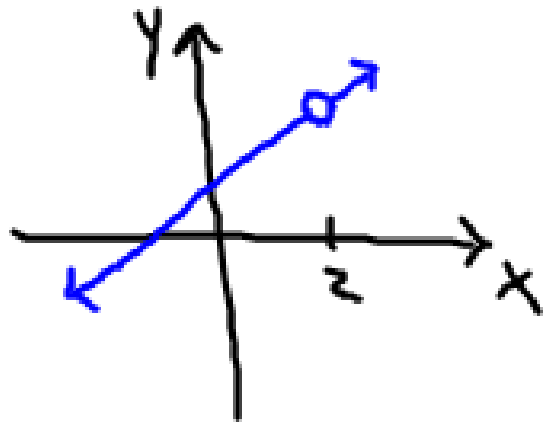
What are the graphical implications of this result?

**What does the graph of this function look like?**

$$f(x)=\frac{x^2-x-2}{x-2} = \frac{(x-2)(x+1)}{x-2}$$

$f(x) = x+1$

$\Rightarrow$ linear graph with a hole at $x=2$




POD = point of discontinuity

(a.k.a a "hole")

$x-2=0$

$x=2$

## Summary: Holes, Vertical Asymptotes, and x-intercepts for Rational Functions

1. If the numerator contains a factor not found in the denominator, that factor will produce an x-intercept for the function.

**Unique factors in the numerator produce x-intercepts or zeros**

## Summary: Holes, Vertical Asymptotes, and x-intercepts for Rational Functions

1. If the numerator contains a factor not found in the denominator, that factor will produce an x-intercept for the function.

2. If the denominator contains a factor not found in the numerator, that factor will produce a vertical asymptote for the function.

**Unique factors in the denominator produce vertical asymptotes**

## Summary: Holes, Vertical Asymptotes, and x-intercepts for Rational Functions

1. If the numerator contains a factor not found in the denominator, that factor will produce an x-intercept for the function.

2. If the denominator contains a factor not found in the numerator, that factor will produce a vertical asymptote for the function.

3. If a factor is found in both the numerator and denominator, you need to consider the following:

   a. If the factor occurs equally frequently or more frequently in the numerator than in the denominator, the factor will produce a POD (hole) in the graph.

   b. If the factor occurs more frequently in the denominator than in the numerator, the factor will produce a vertical asymptote for the graph.

Ex.1 Find any values for $x$ which the function is: a) 0 b) undefined c) indeterminate

$$f(x)=\frac{(x-2)(x+5)}{x(x-2)(x+6)}$$

a) $f(x)=0$ → look for unique factors in num.

$x+5=0$

$x=-5$

b) $f(x)=$ und. → look for unique factors in denom.

$x=0$ $\quad x+6=0$

$x=-6$

c) $f(x) = \frac{0}{0}$ indeterminate → look for factors common to both the num. and denom.

$x - 2 = 0$

$x = 2$ ⇒ hole or POD

Ex.1 Find any values for $x$ which the function is: a) 0 b) undefined c) indeterminate

$$f(x)=\frac{x^4-8x}{x^2-4}=\frac{x(x^3-8)}{(x+2)(x-2)}=\frac{x(x-2)(x^2+2x+4)}{(x+2)(x-2)}$$

a) $f(x)=0$ $\quad x=0$

$x^2+2x+4=0$

$D=b^2-4ac=2^2-4(1)(4)$

$D=4-16=-12$

D is neg $\Rightarrow$ no soln

b) $f(x)=$ und.

$x+2=0$

$x=-2$

c) $f(x)=\frac{0}{0}$ $\quad (x-2)=0$

$x=2$ (hole)

**Your Turn #7**

Consider the following functions. Is there a hole, a vertical asymptote line, or neither at $x = 1$ ?

(a) $f(x) = x - 1$ neither

(b) $f(x) = \dfrac{x^2}{x-1}$ V.A.

(c) $f(x) = \dfrac{x(x-1)^2}{x-1}$ hole

(d) $f(x) = \dfrac{x(x-1)}{(x-1)^3}$ V.A.

## *Assignment:*

***Text pg. 24 - 25,***

***#1 - 9, 13, 15, 17***