Probability and Odds (Sec. 5.2)

Learning Targets:

1. Understanding "odds in favour" of an event occurring and how to calculate it.
2. Understanding "odds against" an event occurring and how to calculate it.
3. Understanding how odds are related to probabilities.
4. Calculating odds based on given probabilities.
5. Calculating probabilities based on given odds.
6. Interpreting odds and/or probabilities to make decisions.

"Odds" are not exactly the same thing as probabilities

Probabilities are expressed as fractions (or decimals or percents).

Odds are always expressed as ratios in the form a : b

The two numbers that are found in the probability fraction are not the same two numbers that would be found in the odds ratio.

Calculating Odds:

There are two kinds of odds that can be calculated:

odds in favour

odds against

In order to calculate either of these odds regarding an event A, you need to know:

the probability of event A occurring: $P(A)$

the probability of event A not occurring: $P(A')$

OR

the number of favourable outcomes for event A: $n(A)$

the number of unfavourable outcomes for event A: $n(A')$

Calculating Odds:

To calculate odds in favour: $P(A) : P(A')$ or $n(A) : n(A')$

To calculate odds against: $P(A') : P(A)$ or $n(A') : n(A)$

Example #1:

If you roll a 6-sided die one time, calculate the odds in favour of rolling a 5 and the odds against rolling a 5.

Using "favourable" and "unfavourable" outcomes:

$\{5\}$ $\{1,2,3,4,6\}$

odds in favour = 1 : 5 odds against = 5 : 1

Example #2:

If you draw a single card from a standard deck of 52 cards, calculate the odds in favour of getting a heart.

n(fav) : n(unfav)

$13 : 39 \Rightarrow 1 : 3$ (÷13 : ÷13)

all of the hearts

all of the cards that aren't hearts

Example #3: $\{HH, HT, TH, TT\}$

Suppose you flip a quarter twice.

What are the odds in favour of getting heads both times?

$$P(A) : P(A') = \frac{1}{4} : \frac{3}{4} = 1 : 3$$

What are the odds against getting one of each?

$$P(B') : P(B) = \frac{2}{4} : \frac{2}{4} = 1 : 1$$

You Try:

Using a standard deck of 52 cards, you draw one card:

Determine the odds in favour of getting a king.

$4 : 48 \Rightarrow 1 : 12$

Determining odds from probabilities:

If given P(A), calculate P(A').

Then form the ratio of:

P(A) : P(A') for "odds in favour"

P(A') : P(A) for "odds against"

Ratios should then be reduced and/or simplified:

- **no fractions or decimals**
- **divide out common factors**

Example #4:

Suppose that the probability of an event A occurring is 2/5. $P(A') = 1 - P(A)$

a) Calculate the **odds in favour** of event A occurring

$\frac{2}{5} : \frac{3}{5} \Rightarrow 2:3$

b) What are the **odds against** event A occurring?

3:2

Example #5:

P(A)

The weatherman says that there is a 70% chance of rain tomorrow. What are the odds against rain tomorrow?

30% : 70% = 3 : 7

Determining probability from odds

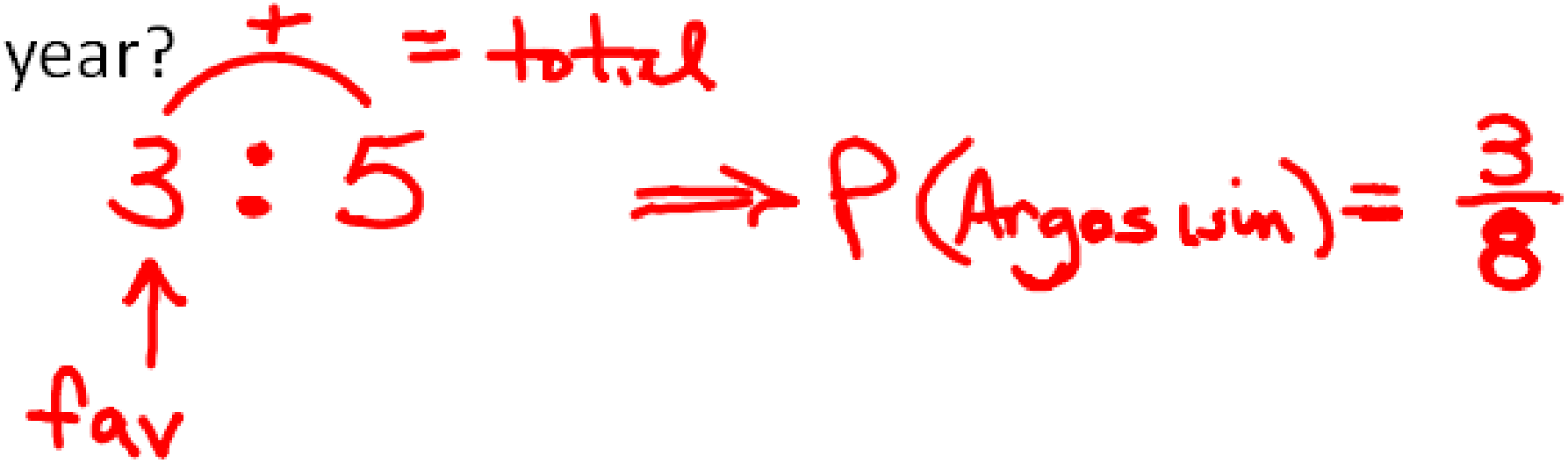
If the **odds in favour** of event A is $m:n$, then:

→ the probability of event A occurring can be calculated as follows: $P(A) = \frac{m}{m+n}$

→ The probability of event A not occurring can be calculated as follows: $P(A') = \frac{n}{m+n}$

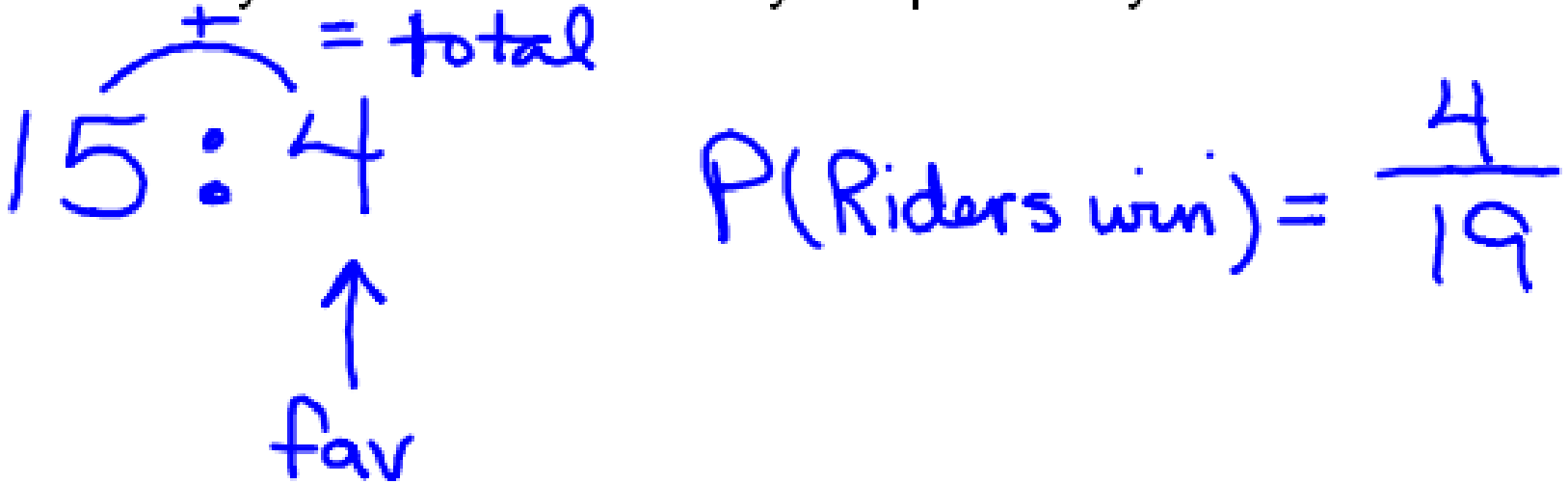
Example #6

Suppose the ***odds in favour*** of the Argos winning the Grey Cup next year is **3:5**, what is the probability that they will win the Grey Cup next year?



Example #7

Suppose the ***odds against*** the Riders winning the Grey Cup next year is **15:4**, what is the probability that they will win the Grey Cup next year?



Decision making based on odds and/or probability

A group of students are holding a charity carnival to support a local animal shelter. They want to add one more game and need to choose between game A and game B. The odds against winning game A are 11:3 and the odds against winning game B are 17:5. The goal is to raise as much money as possible for the animal shelter. Which game should they choose? Assume that people are equally likely to play either game.

→ Choose the game with the higher probability of losing.

Game A → higher prob. of losing
→ lower prob. of winning

OA = 11 : 3

$P(\text{losing}) = \frac{11}{14} = 78.6\%$

$P(\text{winning}) = \frac{3}{14} = 21.4\%$

Game B

OA = 17 : 5

$P(\text{losing}) = \frac{17}{22} = 77.3\%$

$P(\text{winning}) = \frac{5}{22} = 22.7\%$

Check your understanding:

pg. 310 - 312:
#1 - 16