Unit 3

Roots and Powers

(Chapter 4)

BUILDING ON

- determining the square root of a positive rational number
- applying the exponent laws for powers with integral bases and whole number exponents

Unit Overview:

BIG IDEAS

- Any number that can be written as the fraction $\frac{m}{n}$, $n \neq 0$, where m and n are integers, is rational.
- Exponents can be used to represent roots and reciprocals of rational numbers.
- The exponent laws can be extended to include powers with rational and variable bases, and rational exponents.

NEW VOCABULARY

irrational number

real number

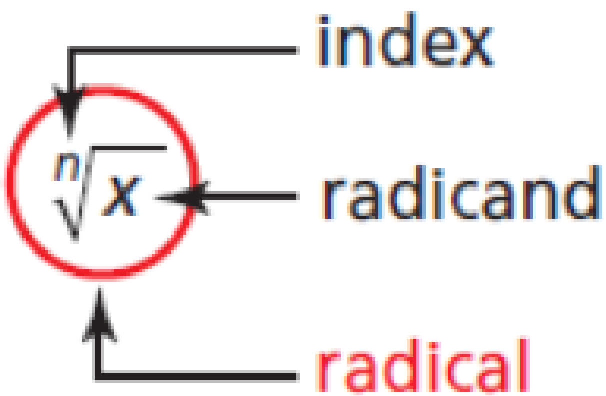
entire radical

mixed radical

Lesson 4.1 / 4.2:

Radicals and Irrational Numbers

Radicals - terminology



The index of a square root is "2" but is rarely written in the radical.

Positive radicands are found in radicals of any index, but a radicand can only be negative when the index is odd.

Radicals - calculator buttons

Square root: $\sqrt{\ }$

"other" roots: $\sqrt[\square]{\ }$ $\quad y^{1/x}$ $\quad x^{1/y}$

$\sqrt[x]{\ }$ $\quad \sqrt[x]{y}$

Example:

Evaluate the following using your calculator:

$\sqrt{\ }$ 8 1 = $\qquad$ $\sqrt{\ }(64 \div 9) =$

$\sqrt{81} = 9 \qquad \sqrt{0.25} = 0.5 \qquad \sqrt{\frac{64}{9}} = 2.\overline{6} = \frac{8}{3}$

8 1 $\sqrt{\ }$ $\qquad$ $64 \div 9 = \sqrt{\ }$

$\sqrt[3]{-27} = -3 \qquad \sqrt[3]{8} = 2 \qquad \sqrt[4]{625} = 5 \qquad \sqrt[5]{-1} = -1 \qquad \sqrt[5]{32}$ $\quad$ 2

3 $\sqrt[x]{\ }$ $-27 =$

Radicals such as these are considered RATIONAL NUMBERS because we could evaluate them exactly (we didn't need to round).

$\sqrt{81}$ $\sqrt{0.25}$ $\sqrt{\frac{64}{9}}$

$\sqrt[3]{-27}$ $\sqrt[3]{8}$ $\sqrt[4]{625}$ $\sqrt[5]{-1}$ $\sqrt[5]{32}$

- Any number that can be written as the fraction $\frac{m}{n}$, $n \neq 0$, where m and n are integers, is rational.

- ratios of integers
- their decimal form either terminates or repeats

Example:

Evaluate the following using your calculator:

$\sqrt{5}$
$= 2.236067...$

$\sqrt[3]{20}$
$= 2.71441...$

$\sqrt[3]{-55}$
$= -3.80295...$

$\sqrt[4]{12}$
$= 1.861209...$

$\sqrt[5]{76}$
$= 2.377730...$

$\sqrt[5]{-16}$
$= -1.741101...$

Radicals such as these are considered IRRATIONAL NUMBERS because we can only <u>estimate</u> their value by rounding.

$\sqrt{5}$ $\sqrt[3]{20}$ $\sqrt[3]{-55}$

$\sqrt[4]{12}$ $\sqrt[5]{76}$ $\sqrt[5]{-16}$

Irrational Numbers

An **irrational number** *cannot* be written in the form $\frac{m}{n}$, where m and n are integers, $n \neq 0$. The decimal representation of an irrational number neither terminates nor repeats.

When dealing with integer radicands, irrational numbers will occur any time the:

- radicand of a square root is not a perfect square

- radicand of a cube root is not a perfect cube

- radicand of a fourth root is not a perfect 4th power

- radicand of a fifth root is not a perfect 5th power

etc.

ROOTS	squares	cubes	4th powers	5th powers
x	x^2	x^3	x^4	x^5
1	1	1	1	1
2	4	8	16	32
3	9	27	81	243
4	16	64	256	1,024
5	25	125	625	3,125
6	36	216	1,296	7,776
7	49	343	2,401	16,807
8	64	512	4,096	32,768
9	81	729	6,561	59,049
10	100	1,000	10,000	100,000
11	121	1,331	14,641	161,051
12	144	1,728	20,736	248,832
13	169	2,197	28,561	371,293
14	196	2,744	38,416	537,824
15	225	3,375	50,625	759,375

Example:

Classify each of the following as rational or irrational:

$$\sqrt{\frac{49}{16}}$$ ← both are perfect squares

∴ rational

$$\sqrt[3]{6}$$ ← not a perfect cube

∴ irrational

$\sqrt[3]{-30}$ ← not a perfect cube

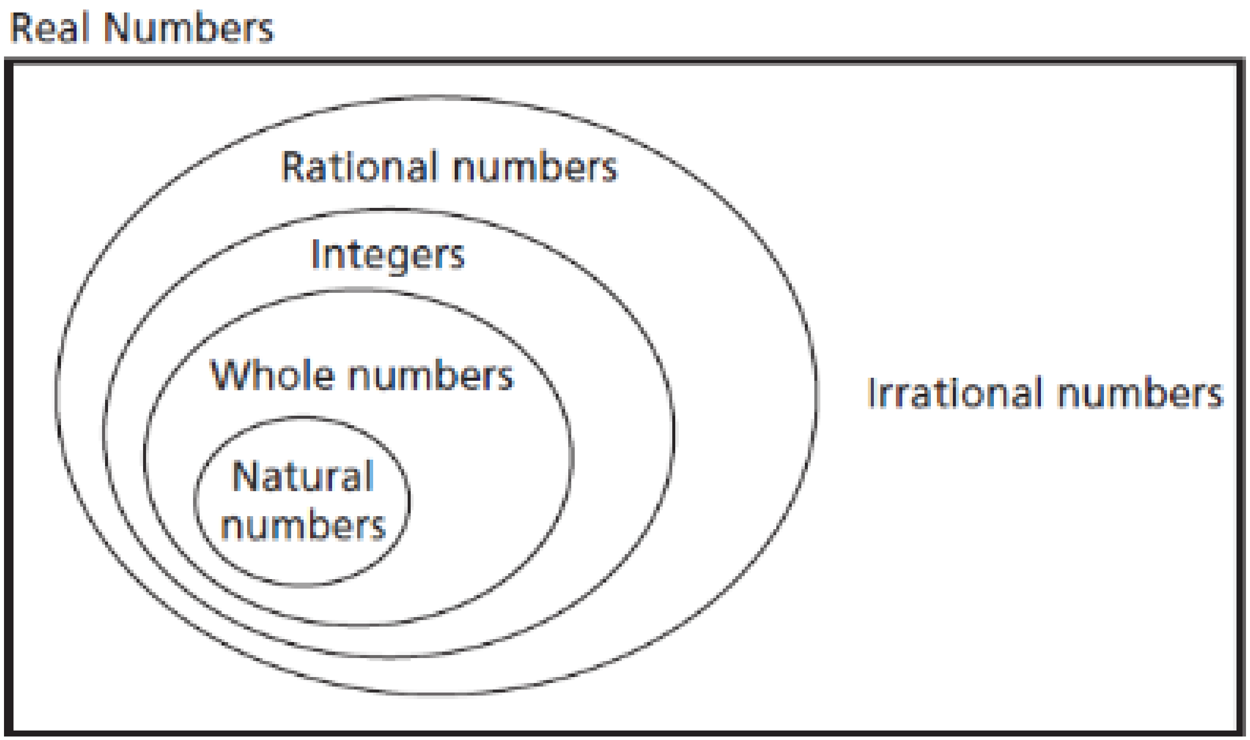
∴ irrational

$\frac{121}{5}$ fraction

∴ rational

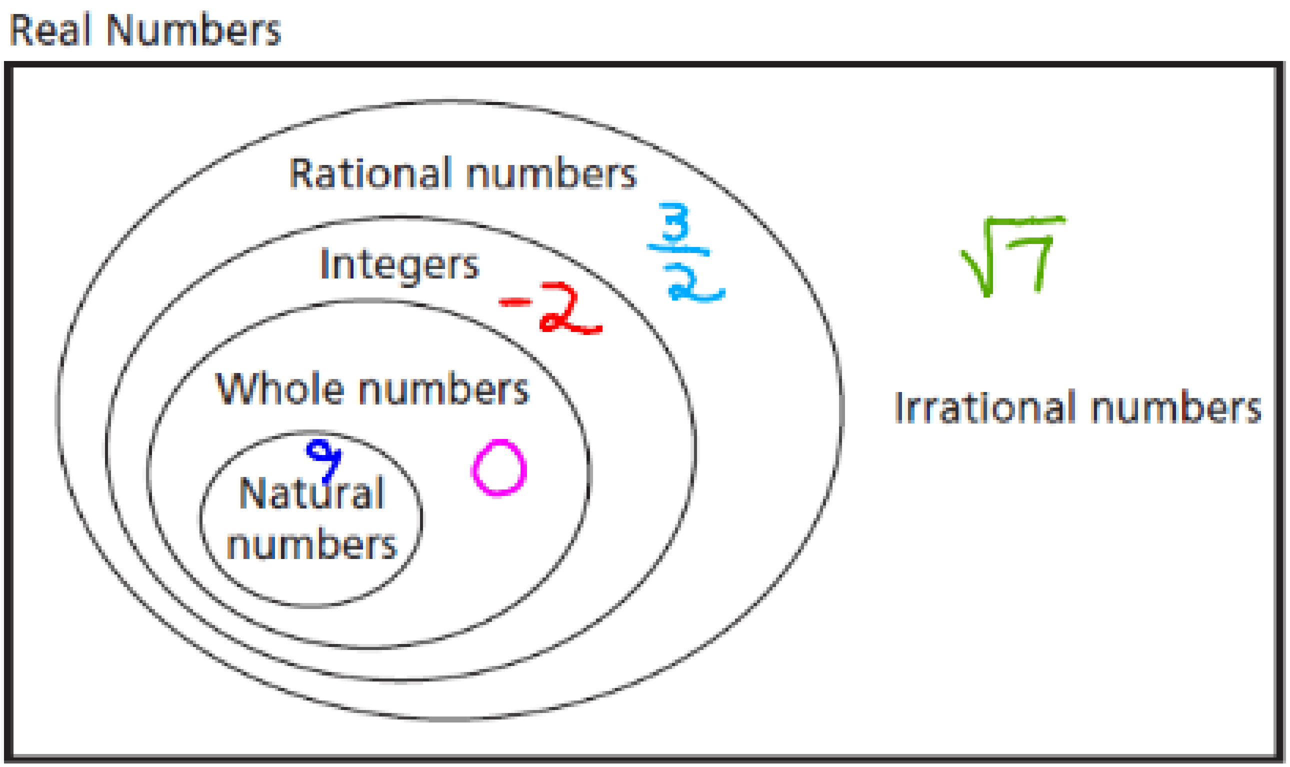
Together, the rational numbers and irrational numbers form the set of **real numbers**.

This diagram shows how these number systems are related.



Sort these numbers into the correct placement in the diagram:

$-2,\quad 9,\quad \sqrt{7},\quad \frac{3}{2},\quad 0$

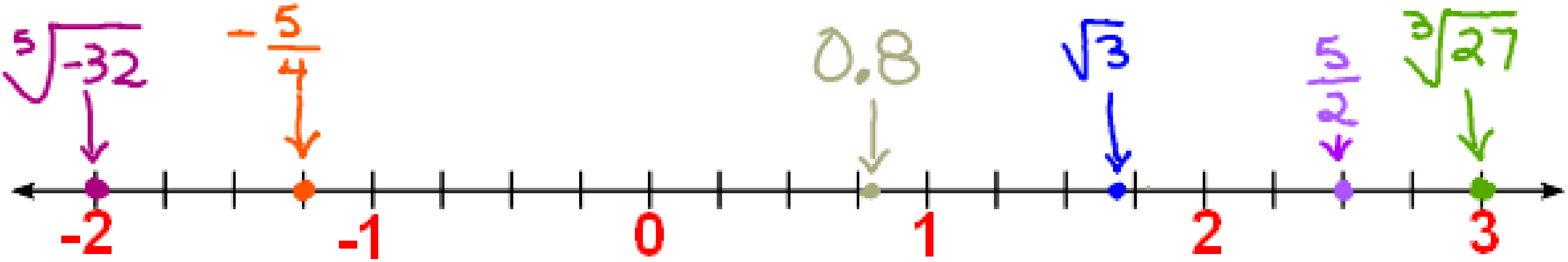


Example:

Order the following by placing them on a number line:

$\sqrt{3}$ $\sqrt[3]{27}$ $\frac{-5}{4}$ $\frac{5}{2}$ $\sqrt[5]{-32}$ 0.8

1.732... 3 -1¼ 2½ -2



Check your understanding:

pg. 206
#1, 2, 4, 6

pg. 211
#3, 4, 5, 9, 10, 11, 12