Lesson #3: Evaluating Limits - day 2 (Section 3.3)

Learning Targets:

i) Evaluating limits of rational functions that can be simplified by factoring to remove a common factor

ii) Evaluating limits of other types of functions using simplification to remove a common factor

iii) Evaluating limits of functions involving radical expressions by using a rationalizing process

Recall: direct substitution can only be used to evaluate limits of functions when direct substitution does not result in zero in the denominator (***undefined***) or zero in both the numerator and denominator (***indeterminate***).

In many indeterminate cases, there are steps we can take to eliminate the problem by **removing a common factor** from the numerator and denominator and then proceed with direct substitution. These strategies are:

- Factoring
- Simplifying
- Rationalizing

Evaluating Limits by Factoring

Determine each of the following limits:

a) $$\lim_{x \to 6} \frac{x^2 - 7x + 6}{x^2 - 36}$$

Direct substitution will result in $\frac{0}{0}$

$$= \lim_{x \to 6} \frac{(x-6)(x-1)}{(x-6)(x+6)}$$

Factor and cancel common factor

$$= \lim_{x \to 6} \frac{x-1}{x+6}$$

We can now proceed with direct substitution

$$= \frac{6-1}{6+6} = \frac{5}{12}$$

b) $$\lim_{x\to 2} \frac{x^3-8}{x^2-3x+2}$$ Direct substitution will result in $\frac{0}{0}$

$$= \lim_{x\to 2} \frac{(x-2)(x^2+2x+4)}{(x-2)(x-1)}$$ Factor and cancel common factor

$$= \lim_{x\to 2} \frac{x^2+2x+4}{x-1}$$ We can now proceed with direct substitution

$$= \frac{2^2+2(2)+4}{2-1} = \frac{12}{1} = 12$$

Evaluating Limits by Simplifying

Determine each of the following limits:

a) $$\lim_{h\to 0} \frac{(2+h)^2-4}{h}$$

Direct substitution will result in $\frac{0}{0}$

$$= \lim_{h\to 0} \frac{4 + 4h + h^2 - 4}{h}$$

Expand in order to simplify

$$= \lim_{h\to 0} \frac{h(4+h)}{h}$$

Factor in order to simplify

We can now proceed with direct substitution

$$= \lim_{h\to 0}(4+h) \quad = 4 + 0 = 4$$

b) $$\lim_{x\to 0} \frac{\frac{1}{x+3} - \frac{1}{3}}{2x}$$

Direct substitution will result in $\frac{0}{0}$

$$= \lim_{x\to 0} \frac{\left(\frac{1}{x+3} - \frac{1}{3}\right)(3)(x+3)}{2x(3)(x+3)}$$

Multiply by the LCD of the numerator in order to simplify

$$= \lim_{x\to 0} \frac{3-(x+3)}{6x(x+3)}$$

$$= \lim_{x\to 0} \frac{3-x-3}{6x(x+3)}$$

Terms cancel

$$= \lim_{x\to 0} \frac{-x}{6x(x+3)}$$

The factor cancels out

$$= \lim_{x\to 0} \frac{-1}{6(x+3)} = \frac{-1}{6(0+3)} = \frac{-1}{18}$$

We can now proceed with direct substitution

Evaluating Limits by Rationalizing

Determine each of the following limits:

a) $$\lim_{x \to 0} \frac{\sqrt{x+1} - 1}{x}$$

Direct substitution will result in $\frac{0}{0}$

$$= \lim_{x \to 0} \frac{\sqrt{x+1}-1}{x} \cdot \frac{\sqrt{x+1}+1}{\sqrt{x+1}+1}$$

Rationalize the numerator by multiplying by the conjugate of the numerator in order to simplify

$$= \lim_{x \to 0} \frac{x+1+\sqrt{x+1}-\sqrt{x+1}-1}{x(\sqrt{x+1})+1)}$$

Terms cancel out

$$= \lim_{x \to 0} \frac{x}{x(\sqrt{x+1})+1)}$$

The factor cancels out

$$= \lim_{x \to 0} \frac{1}{\sqrt{x+1} + 1}$$

We can now proceed with direct substitution

$$= \frac{1}{\sqrt{0+1}+1}$$

$$= \frac{1}{2}$$

b) $\lim_{r \to 6} \frac{\sqrt{3+r} - 3}{r-6}$ **Direct substitution will result in $\frac{0}{0}$**

$= \lim_{r \to 6} \frac{\sqrt{3+r} - 3}{r-6} \cdot \frac{\sqrt{3+r} + 3}{\sqrt{3+r} + 3}$ **Rationalize the numerator by multiplying by the conjugate of the numerator in order to simplify**

$= \lim_{r \to 6} \frac{3+r+3\sqrt{3+r} - 3\sqrt{3+r} - 9}{(r-6)(\sqrt{3+r}+3)}$ **Terms cancel out**

$= \lim_{r \to 6} \frac{r-6}{(r-6)(\sqrt{3+r}+3)}$ **The factor cancels out**

$= \lim_{r \to 6} \frac{1}{\sqrt{3+r} + 3}$

$$= \lim_{x \to 6} \frac{1}{\sqrt{3+r}+3}$$

We can now proceed with direct substitution

$$= \frac{1}{\sqrt{3+6}+3}$$

$$= \frac{1}{\sqrt{9}+3}$$

$$= \frac{1}{6}$$

Assignment:

Text pg. 138,

#19 - 24, 28 - 37