Lesson #3: Elementary Differentiation Rules
(Section 4.5)

Learning Targets:

i) Finding derivatives using the **constant rule**
ii) Finding derivatives using the **power rule**
iii) Finding derivatives using the **general power rule**
iv) Finding derivatives using the **constant multiple rule**
v) Finding derivatives using the **sum and difference rule**
vi) Determining the equation of a tangent line to a function at a specific point

It would be time consuming and tedious if we had to always compute derivatives from the definition of a derivative.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

The **elementary differentiation rules** will provide us with simpler rules to follow in order to calculate derivatives without having to resort to the limit definition.

1. Constant Rule

If f is a contsant function, $f(x) = c$,
then $f'(x) = 0$.

$$\boxed{\frac{d(c)}{dx} = 0}$$

Examples:

$$\boxed{a)\ f(x) = 7}$$

$$f'(x) = 0$$

$b)\ y = \pi$

$y' = 0$

$$c)\ \frac{d}{dx}(-4.5) = 0$$

d) $f(x) = 2^{10}$

$f'(x) = 0$

e) $y = -\frac{100}{3}$

$y' = 0$

f) $\frac{d}{dx}\left(\sqrt{3}\right) = 0$

2. Power Rule

If $f(x) = x^n$, where n is a positive integer,
then $f'(x) = nx^{n-1}$.

$$\frac{d(x^n)}{dx} = nx^{n-1}$$

Examples:

$$\boxed{a)\ f(x) = x^7}$$

$$f'(x) = 7x^6$$

$b)\ y = x^{100}$

$y' = 100x^{99}$

c) $\frac{d}{du}\left(u^9\right) = 9u^8$

Ex.1 Find the **equation** of the **tangent line** to the curve $y = x^6$ at the point (-2,64).

$m_{tan} = y' = 6x^5$

slope at $x = -2$ $= 6(-2)^5$

$= 6(-32)$

$= -192$

$m_{tan} = -192 \quad (x_1, y_1) = (-2, 64)$

$$y - y_1 = m(x - x_1)$$

$$y - 64 = -192(x + 2)$$

$$y - 64 = -192x - 384$$

$$y = -192x - 320$$

3. General Power Rule

If n is any real number, then

$$\frac{d\left(x^n\right)}{dx} = nx^{n-1}$$

With the general power rule we can address variables in denominators by switching to negative exponents and address radicals by switching to fractional exponents:

Negative exponents: $\frac{1}{x^a} = x^{-a}$

Radicals: $\sqrt[n]{x} = x^{\frac{1}{n}}$ $\sqrt[n]{x^m} = x^{\frac{m}{n}}$

Examples:

$$\boxed{a)\ f(x) = \frac{1}{x^3}} = x^{-3}$$

$$f'(x) = -3x^{-4} = \frac{-3}{x^4}$$

We try to avoid negative exponents in our derivatives.

$$\boxed{b)\ y = \sqrt{x}} = x^{1/2}$$

$$y' = \frac{1}{2}x^{-1/2} = \frac{1}{2x^{1/2}}$$

We try to avoid negative exponents in our derivatives.

4. Constant Multiple Rule

If $g(x) = cf(x)$, then
$g'(x) = cf'(x)$.

Examples:

$$a)\ f(x) = 8x^3$$

$$f'(x) = 8(3x^2)$$

$$= 24x^2$$

$$b)\ y = 6x^{\frac{8}{3}}$$

$$y' = 6\left(\frac{8}{3}x^{5/3}\right)$$

$$= 16x^{5/3}$$

$$g(x) = \frac{5}{x^2} = 5x^{-2}$$

$$g'(x) = 5(-2x^{-3})$$

$$= -10x^{-3}$$

$$= -\frac{10}{x^3}$$

$$y = \frac{1}{4x^3} = \frac{1}{4}x^{-3}$$

$$y' = \frac{1}{4}(-3x^{-4})$$

$$y' = -\frac{3}{4}x^{-4}$$

$$y' = \frac{-3}{4x^4}$$

$$f(x) = -3x^{-7}$$

$$f'(x) = -3(-7x^{-8})$$

$$f'(x) = 21x^{-8}$$

$$f'(x) = \frac{21}{x^8}$$

$$y = \frac{11}{x} = 11x^{-1}$$

$$y' = 11(-1x^{-2})$$

$$y' = -11x^{-2}$$

$$y' = \frac{-11}{x^2}$$

$$f(x) = -\frac{2}{x^8} = -2x^{-8}$$

$$f'(x) = -2(-8x^{-9})$$

$$f'(x) = 16x^{-9}$$

$$f'(x) = \frac{16}{x^9}$$

$$y = 4\sqrt{x} = 4x^{1/2}$$

$$y' = 4\left(\frac{1}{2}x^{-1/2}\right)$$

$$y' = 2x^{-1/2}$$

$$y' = \frac{2}{x^{1/2}}$$

Ex. 2 At what points on the hyperbola $xy = 12$ is the tangent line parallel to the line $3x + y = 0$.

Rewrite: $xy = 12$

$y = 12/x$

$y = 12x^{-1}$

Find y':

$y' = -12x^{-2}$

$y' = -12/x^2$

Determine the slope of the given line:

$3x + y = 0$

$y = -3x$ slope $= -3$

The tangent line will be parallel to the given line when their slopes are equal:

$-12/x^2 = -3$ Solve for x

$$\frac{-12}{x^2} = -3$$

$$-3x^2 = -12$$

$$x^2 = 4$$

$$x = \pm 2$$

These are the x-coordinates of the points we are trying to find

Use the equation of the hyperbola to calculate the y-coordinates:

$y = 12/x$ $x = 2$, $x = -2$

$y = 12/2$ $y = 12/-2$

$y = 6$ $y = -6$

The two points are (2, 6) and (-2, -6)

5. Sum and Difference Rule

Sum Rule

If both $f(x)$ and $g(x)$ are differentiable functions, then if $y = f(x) + g(x)$, then $y' = f'(x) + g'(x)$

Difference Rule

If both $f(x)$ and $g(x)$ are differentiable functions, then if $y = f(x) - g(x)$, then $y' = f'(x) - g'(x)$

These rules will allow us to differentiate functions with multiple terms on a term by term basis.

Examples:

$$f(x) = 6x^2 + 7x$$

$$f'(x) = 12x + 7$$

$$y = 12x^2 - 3x$$

$$y' = 24x - 3$$

Example #3:

Differentiate each of the following functions by first writing them as the sum/difference of terms in the form cx^n and then applying the appropriate elementary differentiation rules.

a) $f(x) = (5x - 3)^2$ expand

$f(x) = 25x^2 - 30x + 9$

$f'(x) = 50x - 30$

b) $y = (x+4)(2x-5)$ expand

$y = 2x^2 + 3x - 20$

$y' = 4x + 3$

c) $y = \dfrac{(3x+5)(3x-5)}{x^5}$

expand in the numerator

$$y = \frac{9x^2 - 25}{x^5}$$

convert to cx^n form

$$y = \frac{9x^2}{x^5} - \frac{25}{x^5}$$

$$y = 9x^{-3} - 25x^{-5}$$

$$y = 9x^{-3} - 25x^{-5}$$

$$y' = -27x^{-4} + 125x^{-6}$$

$$y' = \frac{-27x^{2}}{x^{4}x^{2}} + \frac{125}{x^{6}}$$

$$y' = \frac{-27x^{2} + 125}{x^{6}}$$

d) $$f(x) = \sqrt{\frac{x}{3}} - \frac{2}{\sqrt{x}} + 6$$

$$f(x) = \frac{1}{\sqrt{3}} x^{1/2} - 2x^{-1/2} + 6$$

$$f'(x) = \frac{1}{2\sqrt{3}} x^{-1/2} + x^{-3/2}$$

$$f'(x) = \frac{1}{2\sqrt{3x}} + \frac{1}{\sqrt{x^3}}$$

Assignment

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Oral Exercises: #1 - 56
(answers posted in OneNote and on weebly)

Written Exercises: #1 - 5, 7, 10