Section 2.2:

Angles Formed by Parallel Lines

Learning Targets (day 1):

1) New terminology:
   - alternate interior angles
   - alternate exterior angles
   - interior angles on the same side of the transversal

2) Proving angle properties and relationships in a 2-column deductive proof format.

2-Column Deductive Proofs:

- **The first column contains statements that we believe are true.**
- **The second column contains the reason or justification for the statement (how do we *know* it's true?)**

Every proof begins with given information, often in the form of a diagram, but sometimes in words and symbols, or a combination of both diagram and words/symbols. Every statement we come up with in the proof is a logical deduction based on the given information and the premises we know to be true about special pairs of angles:

- Corresponding angles are equal when parallel lines are involved
- Vertically opposite angles are equal
- Linear pairs of angles are supplementary

The goal is to provide statements in a step-by-step logical fashion, all building towards being able to state, with certainty, the relationship we are asked to prove as the last line of our proof. If we can provide valid reasons or justifications for each statement along the way, we will have provided a valid proof.

Other mathematical properties we will find useful in proofs include:

- **Substitution property**

If **A** = B and B **+ C** = **D**, then **A + C** = **D**
(*by substitution of A for B*)

- **Transitive property**

If **A** = B and B = **C**, then **A** = **C**
(*this is a special case of substitution*)

Angle Pair Classifications:

Alternate Interior Angles

Alternate Exterior Angles

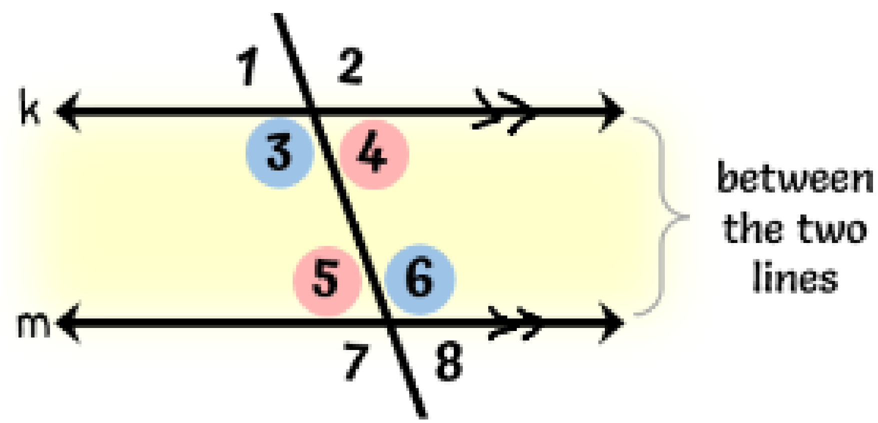
Same-side Interior Angles

Name: Alternate Interior Angles

Abbreviation: ALT INT

Description:
the 2 non-adjacent interior angles on opposite sides of the transversal

Diagram:

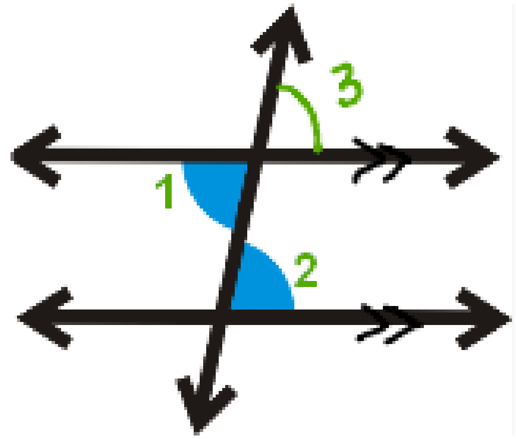


Mathematical relationship:
alternate interior angles are congruent when the lines are parallel

$\angle 3 \cong \angle 6$

$\angle 4 \cong \angle 5$

Proof:



Prove $\angle 1 = \angle 2$

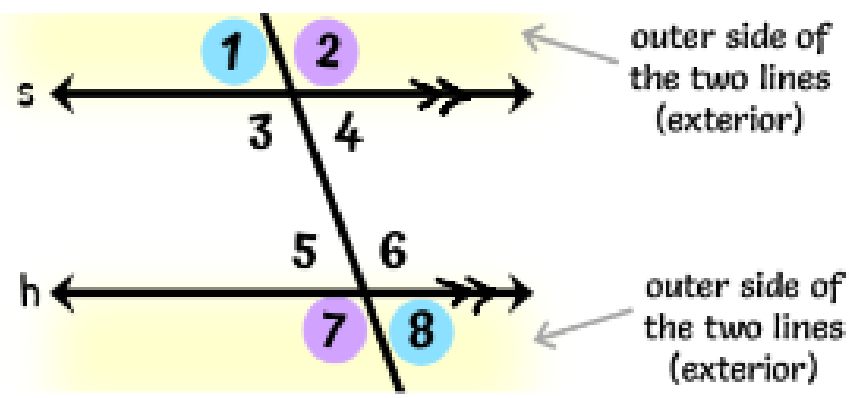
Statement:	**Reason:**
1. line a ∥ line b	1. Given
2. $\angle 1 = \angle 3$	2. VOA
3. $\angle 3 = \angle 2$	3. CORR
4. $\angle 1 = \angle 2$	4. transitive prop.

Name: Alternate Exterior Angles

Abbreviation: ALT EXT

Description:
the 2 non-adjacent exterior angles on opposite sides of the transversal

Diagram:

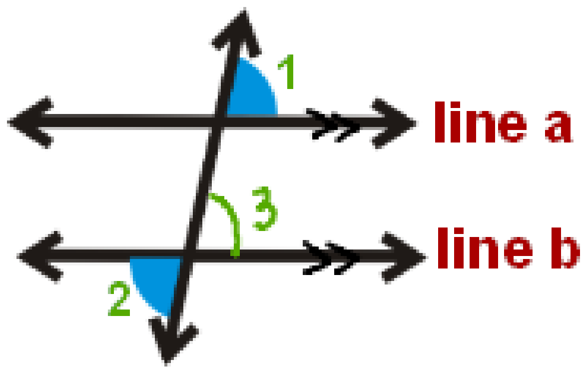


Mathematical relationship:
alternate exterior angles are congruent when the lines are parallel

$\angle 1 \cong \angle 8$

$\angle 2 \cong \angle 7$

Proof:



Prove $\angle 1 = \angle 2$

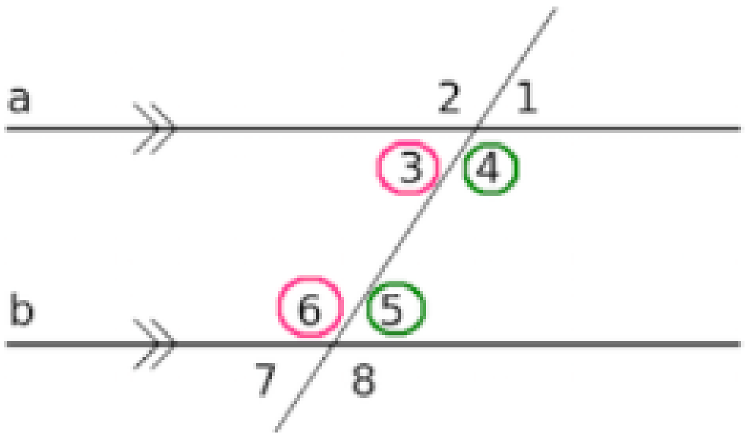
Statement:	**Reason:**
1. line a ∥ line b	1. Given
2. $\angle 1 = \angle 3$	2. CORR
3. $\angle 3 = \angle 2$	3. VOA
4. $\angle 1 = \angle 2$	4. transitive prop.

Name: Same-Side Interior Angles

Abbreviation: SSIA

Description:
the 2 non-adjacent interior angles on the same side of the transversal

Diagram:

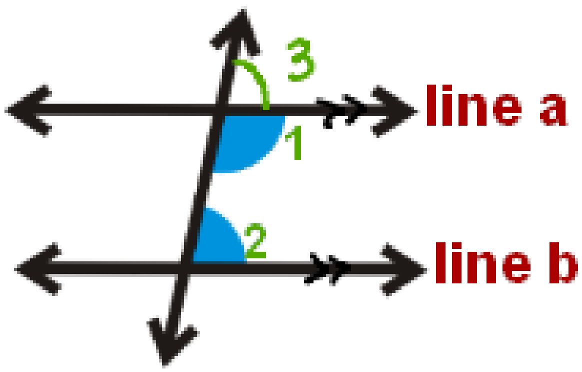


Mathematical relationship:
same-side interior angles are supplementary when the lines are parallel

$\angle 3 + \angle 6 = 180^\circ$

$\angle 4 + \angle 5 = 180^\circ$

Proof:



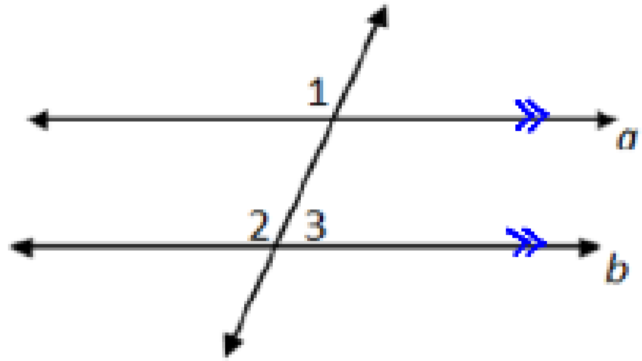
Prove $\angle 1 + \angle 2 = 180°$

Statement:	**Reason:**
1. line a ∥ line b	1. Given
2. $\angle 1 + \angle 3 = 180°$	2. LPA
3. $\angle 3 = \angle 2$	3. CORR
4. $\angle 1 + \angle 2 = 180°$	4. substitution

Example #1

Given: Line a is parallel to Line b

Prove: $\angle 1 + \angle 3 = 180°$

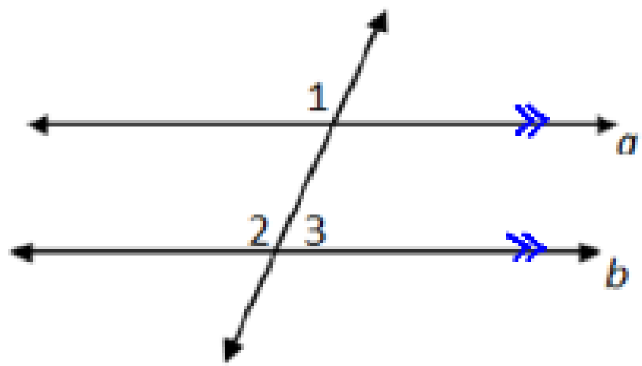


Statement	**Reason**
1.	1.
2.	2.
3.	3.
4.	4.

Example #1

Given: Line $\boldsymbol{a}$ is parallel to Line $\boldsymbol{b}$

Prove: $\angle 1 + \angle 3 = 180°$



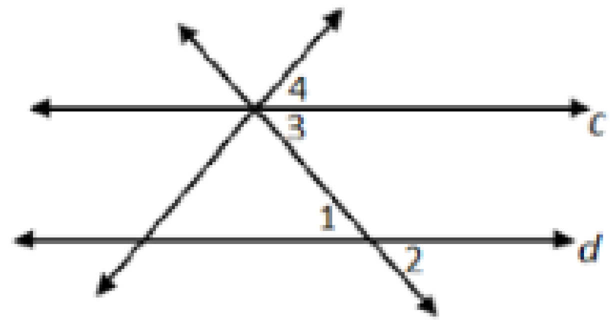
Statement	**Reason**
1. $a \parallel b$	1. Given
2. $\angle 1 = \angle 2$	2. CORR
3. $\angle 2 + \angle 3 = 180°$	3. LPA
4. $\angle 1 + \angle 3 = 180°$	4. substitution

Example #2

Given: Line *c* is parallel to Line *d*

$\angle 4 = \angle 3$

Prove: $\angle 4 = \angle 2$



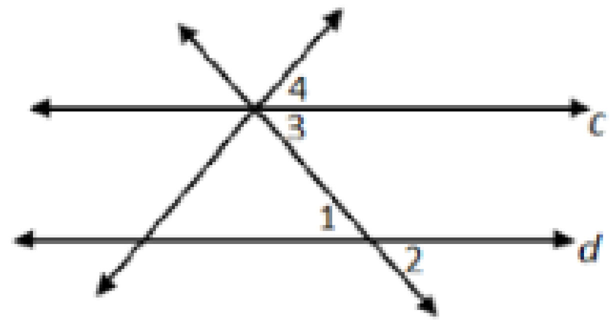
Statement	**Reason**
1.	1.
2.	2.
3.	3.
4.	4.

Example #2

Given: Line c is parallel to Line d

$\angle 4 = \angle 3$

Prove: $\angle 4 = \angle 2$



Statement	**Reason**
1. $c \parallel d$	1. Given
2. $\angle 4 = \angle 3$	2. Given
3. $\angle 3 = \angle 2$	3. CORR
4. $\angle 4 = \angle 2$	4. transitive prop.

Proving Lines are Parallel

When a transversal intersects two lines, if **any** of the following are true, the lines are parallel:

(1) a pair of corresponding angles are equal

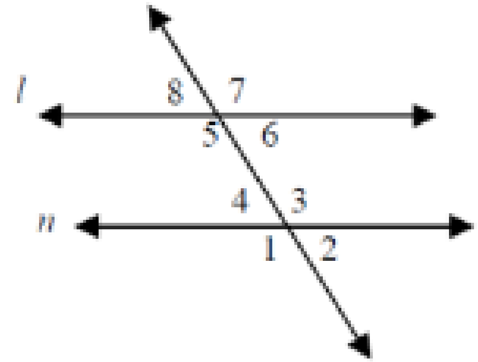
(2) a pair of alternate interior angles are equal

(3) a pair of alternate exterior angles are equal

(4) a pair of same-side interior angles are supplementary

Example #3

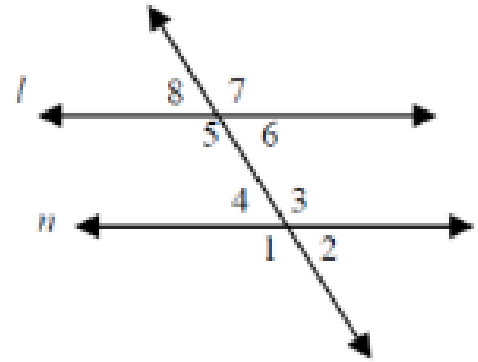
Given: $\angle 8 + \angle 3 = 180°$



Prove: Line l is parallel to Line n

Example #3

Given: $\angle 8 + \angle 3 = 180°$

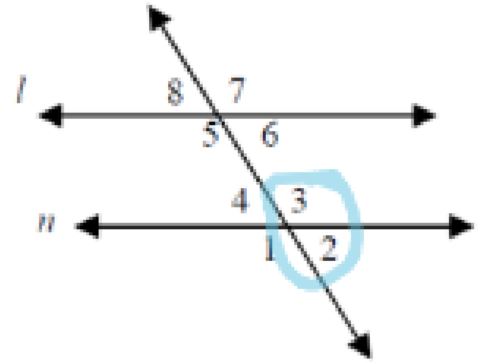


Prove: Line ***l*** is parallel to Line ***n***

Statement	**Reason**
1. $\angle 8 + \angle 3 = 180°$	1. Given

Example #3

Given: $\angle 8 + \angle 3 = 180°$

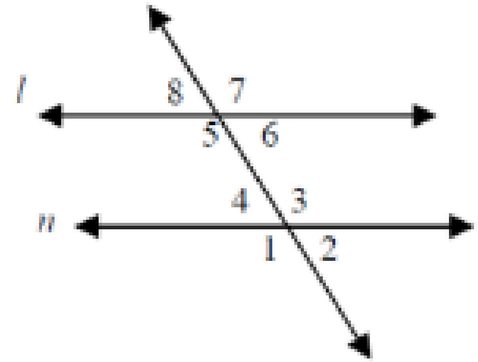


Prove: Line l is parallel to Line n

Statement	Reason
1. $\angle 8 + \angle 3 = 180°$	1. Given
2. $\angle 2 + \angle 3 = 180°$	2. Linear pair

Example #3

Given: $\angle 8 + \angle 3 = 180°$

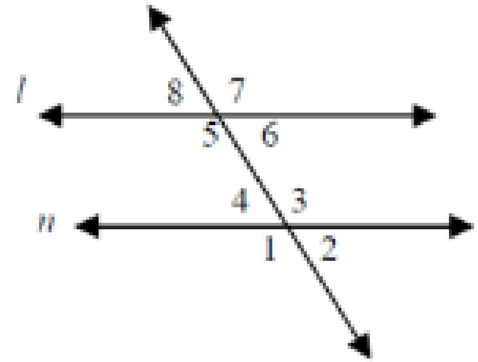


Prove: Line l is parallel to Line n

Statement	Reason
1. $\angle 8 + \angle 3 = 180°$	1. Given
2. $\angle 2 + \angle 3 = 180°$	2. Linear pair
3. $\angle 8 + \angle 3 = \angle 2 + \angle 3$	3. By substitution

Example #3

Given: $\angle 8 + \angle 3 = 180°$

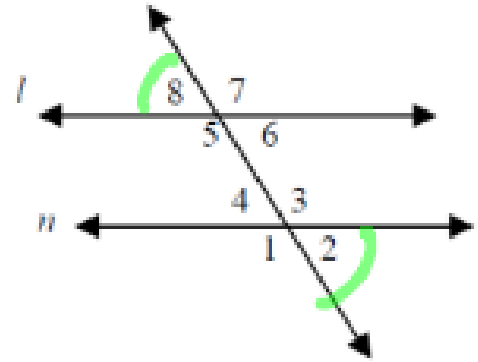


Prove: Line l is parallel to Line n

Statement	Reason
1. $\angle 8 + \angle 3 = 180°$	1. Given
2. $\angle 2 + \angle 3 = 180°$	2. Linear pair
3. $\angle 8 + \angle 3 = \angle 2 + \angle 3$	3. By substitution
4. $\angle 8 = \angle 2$	4. By subtraction

Example #3

Given: $\angle 8 + \angle 3 = 180°$



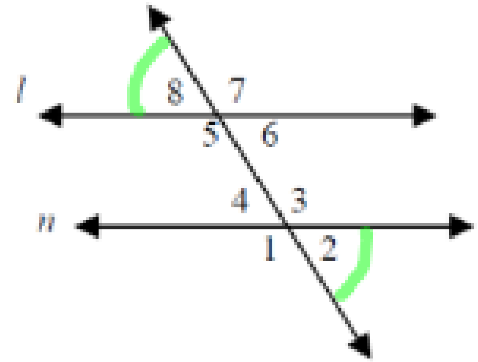
Prove: Line ***l*** is parallel to Line ***n***

Statement	**Reason**
1. $\angle 8 + \angle 3 = 180°$	1. Given
2. $\angle 2 + \angle 3 = 180°$	2. Linear pair
3. $\angle 8 + \angle 3 = \angle 2 + \angle 3$	3. By substitution
4. $\angle 8 = \angle 2$	4. By subtraction

Example #3

Given: $\angle 8 + \angle 3 = 180°$

Prove: Line l is parallel to Line n



Statement	Reason
1. $\angle 8 + \angle 3 = 180°$	1. Given
2. $\angle 2 + \angle 3 = 180°$	2. Linear pair
3. $\angle 8 + \angle 3 = \angle 2 + \angle 3$	3. By substitution
4. $\angle 8 = \angle 2$	4. By subtraction
5. Line l is ‖ to Line n	5. When alt ext $\angle s$ are equal, lines are parallel

Check your understanding:

Handout #1 - 5