Sec. 4.3: Permutations When all Objects are Distinguishable

Learning Targets:

1. Permutation formula: $_nP_r$

2. Understanding when to use the $_nP_r$ permutation strategy

3. Solving a permutation counting problem:

    - using $_nP_r$

    - using $_nP_r$ in a problem involving "**cases**"

    - using $_nP_r$ in a problem involving **conditions or restrictions**

4. Comparing permutations created with and without **repetition**

The Permutation Formula:

When **all n of the objects** in a problem are being used in the arrangements, then there will be **n!** different permutations.

When you are only arranging some of the objects rather than all of them, there is a different formula.

If you have **n distinguishable objects** and you are creating permutations by **using r of the objects at a time**, then the number of permutations will be

$$_nP_r \;=\; \frac{n!}{(n-r)!} \qquad (r \le n)$$

Example:

Suppose 5 people are in a group.

How many different ways can these 5 people be lined up in a row?

How many different ways can you line up *3 of them at a time*?

Evaluating $_nP_r = \frac{n!}{(n-r)!}$ $(r \leq n)$

$_7P_3$

$_8P_5$

$_5P_1$

Rule:

Examples (typical $_nP_r$ problems):

1. How many different displays can be made out of 6 model cars if only 4 of the cars can be used at one time?

2. How many different ways can a president, vice president and secretary be chosen from a group of 12 students?

3. How many three letter arrangements are there of the letters in the word TRIANGLE?
   (hint: imagine the letters are on Scrabble tiles)

4. Matt recently downloaded 40 new songs. How many 12-song playlists can he create if no song gets repeated within a playlist?

What is 0!

0! is the only factorial where the number is not a counting number.

You can't "expand" 0! like other n! expressions so it is difficult to understand how 0! can have a value at all.

The expression $_nP_n$ would mean that you have a group of n objects and you were arranging n of them at a time. This is the same thing as n!

What does the permutation formula $_nP_n$ look like in its expanded form? What does it simplify to?

Rule: **Rule:**

Permutations involving "cases"

Look for an "OR" situation.
Sometimes the phrases "at least" or "at most" will be used.

Example:

Alex needs to create a password for her email account. The password can be made of digits from 0 to 9 and/or any letter of the alphabet. The password is also case-sensitive. The password must be at least 6 characters long, but no more than 8 characters long.

How many different passwords are possible if repetition of characters is not allowed?

Permutations involving CONDITIONS or RESTRICTIONS

Look for: "if….must…" or "if … can't" types of situations
Often will involve a <u>double strategy</u>: FCP where permutations will count the number of choices for each decision.

Example:

Jane has 12 different pairs of shoes and 4 different pairs of boots.
In how many ways can Jane line up her footwear if the shoes must all be grouped together on the left, and the boots together on the right?

Comparing "REPETITION" versus "NO REPETITION ALLOWED"

If we are creating arrangements of r objects taken from a group of n objects, where **repetition <u>is</u> allowed**, the number of permutations is $\boldsymbol{n^r}$

Ex: How many different 3-letter passwords can be created using the 26 letters of the alphabet, allowing repetition?

If we are creating arrangements of r objects taken from a group of n objects, where **repetition <u>is not</u> allowed**, the number of permutations is *nPr*

Ex: How many different 3-letter passwords can be created by using the 26 letters of the alphabet if repetition is not allowed?