Pre-calculus 30

Final Exam Review
Functions and Transformations

55 marks on the Final Exam

Mapping Rule:

(x, y) --> (x/b + h, ay +k)

Used to map functions point-by-point using known coordinates from the base function.

"compression"

Effect of each parameter:

a = vertical stretch (if negative, reflect in the x-axis)

b = horizontal stretch (if negative, reflect in the y-axis)

h = horizontal translation

k = vertical translation

Inverses of Functions:

$\boxed{\text{x and y switch roles}}$

table of values

x	y
-1	1
-2	4
-3	9
0	0
1	1

$\xrightarrow{\text{inv.}}$

x	y
1	-1
4	-2
9	-3
0	0
1	1

<u>Equation</u>

$y = 2x + 7$

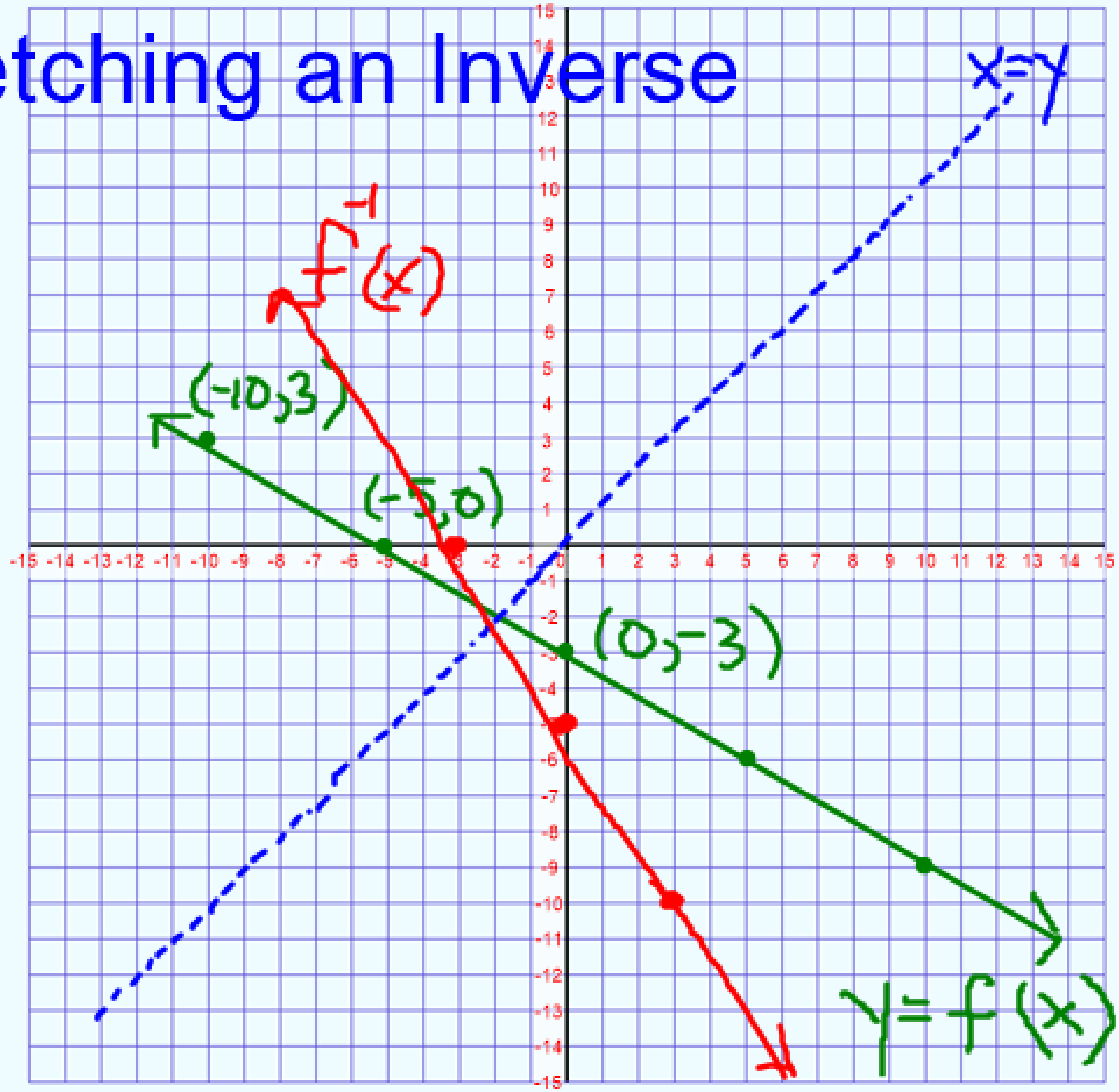
$x = 2y + 7$

$\frac{x-7}{2} = \frac{2y}{2}$

$y = \frac{1}{2}x - 3.5$

Inverse

Sketching an Inverse



Logarithm and Exponent Rules and Properties:

Equal bases rule: $a^x = a^y$, then $x = y$

$\log_b x = y \longrightarrow b^y = x$

$\log_b xy = \log_b x + \log_b y$

$\log_b \frac{x}{y} = \log_b x - \log_b y$

$\log_b x^k = k \log_b x$

Solve an exponential equation with equal bases rule:

$$8^{2x} = 16(2^{x-3})$$

$$(2^3)^{2x} = 2^4(2^{x-3})$$

$$2^{6x} = 2^{x+1}$$

$$6x = x + 1$$

$$5x = 1$$

$$x = 1/5$$

Solve an exponential equation using common logs:

$$4^{x+1} = 122$$

$$\log 4^{x+1} = \log 122$$

$$\frac{(x+1)\cancel{\log 4}}{\cancel{\log 4}} = \frac{\log 122}{\log 4}$$

$$x+1 = \frac{\log 122}{\log 4} \; -1 = 2.465\ldots$$

$$-1$$

Solve a logarithmic equation:

$$\log_4(x-1)+\log_4 2 = 3$$

$$\log_4(2x-2) = 3$$

$$4^3 = 2x-2$$

$$64 = 2x-2$$

$$66 = 2x$$

$$33 = x$$

Function Notation and Operations:

$f(x) = x^2 + 3$ $\qquad$ $g(x) = x - 9$

$fg(x) = (x^2+3)(x-9)$

$f \circ g(x) = (x-9)^2 + 3$

$(f+g)(2) = f(2) + g(2)$

$= 2^2 + 3 + 2 - 9$

$= 4 + 3 + 2 - 9$

$= 0$

$g \circ f(x) = (x^2+3) - 9$

Polynomial Factoring:

- synthetic division
- Remainder Theorem
- Factor Theorem

$$\frac{x^3 - 7x^2 + 3x + 2}{x + 1}$$

$$\begin{array}{r|rrrr}
-1 & 1 & -7 & 3 & 2 \\
 & \downarrow & + & + & + \\
 & & -1 & 8 & -11 \\
\hline
 & 1 & -8 & 11 & \boxed{-9}
\end{array}$$

$$x^2 - 8x + 11 \text{ rem } -9 = \frac{-9}{x+1}$$

Example 4 Sketch the graph of the polynomial function $y = -2x^3 + 6x - 4$

a) The degree 3
b) The leading coefficient -2
c) End Behaviour Q2 → Q4
* d) Zeros $x=1$ $x=-2$
e) Y-intercept -4
f) Intervals the function is positive or negative

$$-2x^3+6x-4 = -2(x+2)(x-1)^2$$

$$-2(x^3-3x+\underline{2})$$

$(x+1)$ $(x+2)$

$(x-1)$ $(x-2)$

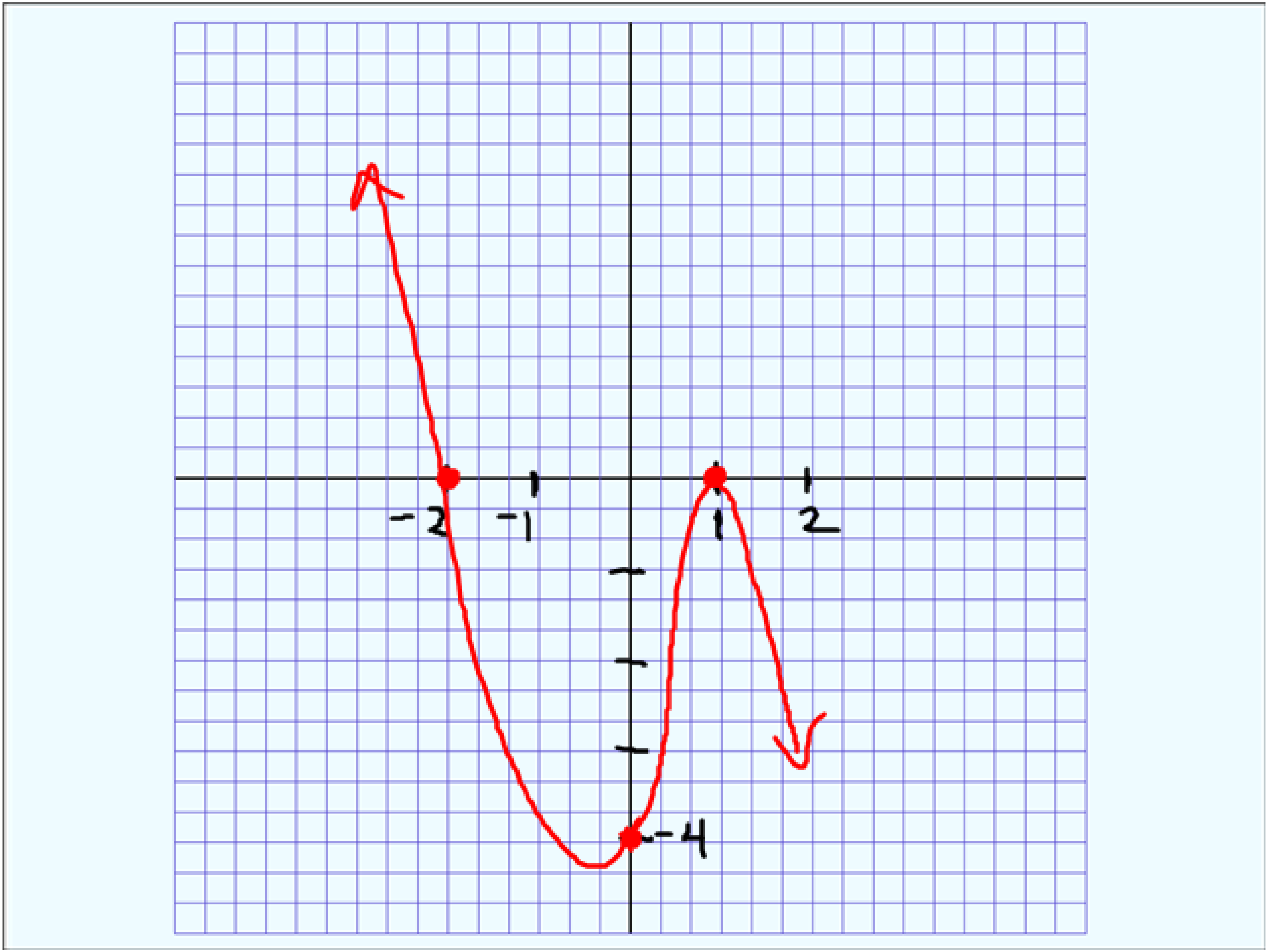
$x=1$

$$1^3-3(1)+2=0$$

$\therefore (x-1)$ is a factor

$$\begin{array}{r|rrrr} 1 & 1 & 0 & -3 & 2 \\ & \downarrow & 1 & 1 & -2 \\ \hline & 1 & 1 & -2 & \boxed{0} \end{array}$$

$\Rightarrow x^2+x-2=(x+2)(x-1)$



-2
-1
1
2
-4

Rational Functions:

- horizontal and vertical asymptotes
- points of discontinuity
- x and y intercepts

$$f(x) = \frac{x-1}{x^2+2x-3} = \frac{\cancel{(x-1)}^{\,1}}{\cancel{(x-1)}(x+3)}$$

$$f(x) = \frac{1}{x+3}$$

$f(x) = \frac{1}{x}$ translated 3 left

$\boxed{VA \quad x=h}$ $\boxed{HA \quad y=k}$

$x=-3$ $y=0$

pod at $x=1$, $y=1/4$ $\Big\}$ $(1, 0.25)$

Rational Function graph