# ***Ch. 6: Derivative Applications***

**Overview:**

In Chapter 6 we will be working with **applications** such as **distance, velocity and acceleration** of an object, **optimization** situations (max/min problems), and **related rates** problems.

***Lesson #1: Rates of Change (Section 6.1)***

***Learning targets:***

i) Average rates of change.
ii) Instantaneous rates of change.

**Average Rate of Change:**

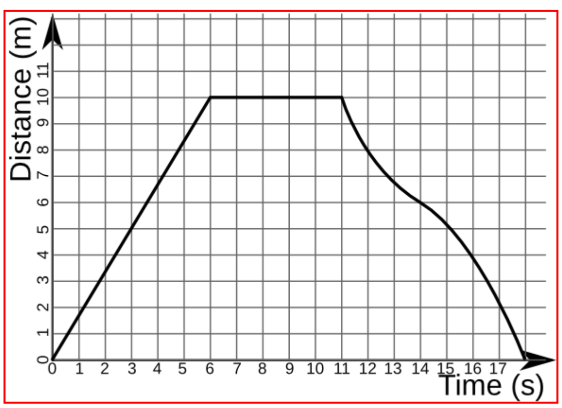
On a graph, the **average rate of change** between two points is the slope of the **secant line** that joins the two points.

**Instantaneous Rate of Change:**

Unlike average rate of change, instantaneous rates of change occur at single points. On a graph, the **instantaneous rate of change** at a point is the slope of the **tangent** line at that point.

**Example #1:**

Use the distance versus time graph shown below to determine the average rate of change from **t = 3 sec** to **t = 12 sec.**



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**Example #2:**

After traveling $x$ kilometres, the temperature under the hood of a car, in degrees Celsius, is given by the function: $\boldsymbol{t(x) = \frac{60x}{\sqrt{x^2+10}} + 20}$

(a) Find the average rate of change in the temperature per km between starting the car and traveling 10 km.

(b) What is the instantaneous rate of change in the temperature per km after traveling 6 km?

***Assignment:*** ***Text pg. 264, #2, 3, 4, 7***

# ***Ch. 6: Derivative Applications***

***Lesson #2: Distance, Velocity and Acceleration (Sec 6.2)***

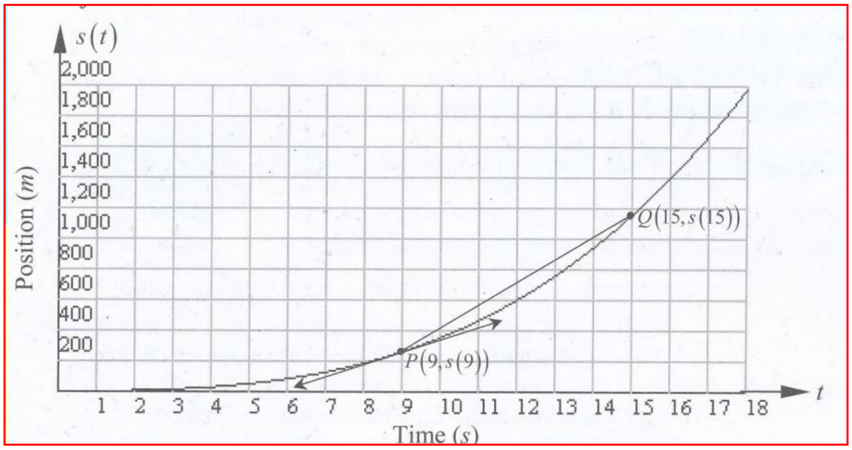
***Learning targets:***

i) Understanding **Position vs. Time** graphs
- the rate of change will have units of distance/time, which is velocity

ii) Understanding **Velocity vs. Time** graphs
- the rate of change will have units of velocity/time, which is acceleration

iii) Understanding the relationships between position, velocity and acceleration:
- Position function: $s(t)$
- Velocity function: $v(t) = s'(t)$
- Acceleration function: $a(t) = v'(t) = s''(t)$

**Position vs. Time Graph:**




*(for this graph, velocity will have units of **m/s**)*

**Position function:**

$\boldsymbol{s(t) = \frac{1}{3}t^3 + 2t}$

**($t$ in secs, $s(t)$ in metres)**

From a position vs. time graph we can find **average velocity**.

**Instantaneous velocity** at a point can be found using the first derivative to find the slope of the tangent line.

Determine the **average velocity** between time **t = 9 sec** and **t = 15 sec**

$v_{ave} = \frac{s(15)-s(9)}{15-9}$

Determine the **instantaneous velocity** at **t = 9 sec**.

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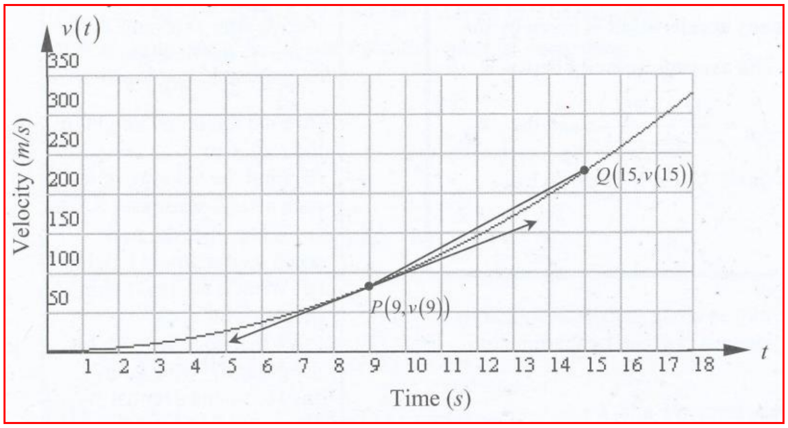
**From a position versus time graph we have the following:**

**Average velocity** $= \frac{change\ in\ distance}{change\ in\ time} \rightarrow v_{ave} = \frac{s(t_2)-s(t_1)}{t_2-t_1}$

**Instantaneous velocity** $= v(t) = \frac{ds}{dt} = s'(t)$

**Velocity vs. Time Graph:**




**Velocity function:**

$v(t) = t^2 + 2$

**($t$ in secs, $v(t)$ in metres/sec)**

From a velocity vs. time graph we can find **average acceleration**.

**Instantaneous acceleration** at a point can be found using the first derivative to find the slope of the tangent line.

*(for this graph, acceleration will have units of* $m/s^2$*)*

Determine the **average acceleration** from time **t = 9 sec** to **t = 15 sec**

$a_{ave} = \frac{v(15)-v(9)}{15-9}$

Determine the **instantaneous acceleration** at **t = 9 sec**.

**From a velocity versus time graph we have the following:**

**Average acceleration** $= \frac{change\ in\ velocity}{change\ in\ time} \rightarrow a_{ave} = \frac{v(t_2)-v(t_1)}{t_2-t_1}$

**Instantaneous velocity** $= a(t) = \frac{dv}{dt} = v'(t) = s''(t)$

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**Example #1:**

A ball rolls along the x-axis so that its position in cm after t seconds is given by $\boldsymbol{s(t) = t^3 + 3t^2, t \geq 0}$

a) Find an expression for the velocity at any time t.

b) Find the velocity at t = 5 sec

c) Find the average velocity between t = 1 sec and t = 5 sec

d) Where is the ball when the velocity is 72 cm/s?

e) Find an expression for the acceleration at any time t.

f) Find the acceleration at t = 1 sec.

g) What is the velocity of the ball when acceleration is 54 cm/s$^2$?

h) Find the average acceleration from t = 1 sec to t = 5 sec.

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**Example #2:**

A rock is thrown upwards from the top of a house and its height above the ground in metres after t seconds is given by the function $\boldsymbol{h(t) = -5t^2 + 40t + 5}$

a) Find the initial height of the rock.

b) Find the velocity of the rock after 2 seconds

c) When does the rock reach its maximum height?

d) What is the maximum height of the rock?

e) When does the rock hit the ground? Round to one decimal place.

f) With what velocity does the rock hit the ground?

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**Example #3:**

Suppose that a bee flies along the x-axis so that its position in feet at time t in seconds is given by

$$s(t) = 2t^3 - 21t^2 + 60t$$

a) Where is the bee when t = 3 sec?

b) What is the velocity of the bee when t = 3 sec?

c) What is the velocity of the bee when t = 6 sec?

d) Find the time interval(s) when the bee is moving to the right.

e) Find the time interval(s) when the bee is moving to the left.

f) Show that the bee has moved a total of 90 feet during the time interval t = 0 to t = 6.

***Assignment:*** ***Text pg. 271, #2 - 7***

*Calculus 30*

# ***Ch. 6: Derivative Applications***

***Lesson #3: Optimization: Number Problems / Perimeter and Area Problems (Sec. 6.3 – day 1)***

***Learning targets:***

i) Using derivatives to solve number optimization problems.
ii) Using derivatives to solve perimeter and area optimization problems.

**Optimization = maximum or minimum**
These problems involve finding the **absolute maximum** or **absolute minimum** for a quantity over a certain interval as dictated by the constraints of the problem.

**Example #1:**

Find two non-negative numbers whose sum is 15 such that the product of one with the square of the other is a maximum.

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**Example #2:**

Two non-negative numbers have a product of 16. Find these numbers if the sum of their squares is to be a minimum. What is the minimum sum of the squares?

# ***Ch. 6: Derivative Applications***

**Rectangular Area / Perimeter Problems:**

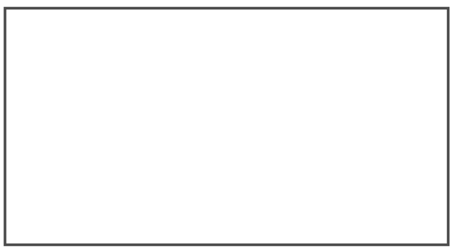
- ➔ Sometimes we will want to maximize area for a given perimeter
- ➔ Sometimes we will want to minimize perimeter for a given area

**Perimeter = 2L + 2W = 2(L + W)**

**Area = LW**

### Example #3:

In a conservation park, a lifeguard has to use 620 metres of marker buoys to rope off a rectangular safe swim area. If one side of the area is the beach, calculate the dimensions of the swimming area so that it is a maximum.



# ***Ch. 6: Derivative Applications***

**Example #4:**

A farmer wants to build a 216 m$^2$ rectangular garden and divide it in half using another fence parallel to one side. What dimensions for the outer rectangle will require the smallest amount of fencing? How much fencing will be used?

***Assignment:*** ***Text pg. 277, #2, 3, 5, 7, 9, 11, 12, 13, 14***

# ***Ch. 6: Derivative Applications***

***Lesson #4: Optimization: Volume and Surface Area Problems (Sec. 6.3 – day 2)***

***Learning targets:***

| | |
|---|---|
| i) | Using derivatives to solve volume and surface area optimization problems. |

**Rectangular Prism Volume / Surface Area Problems:**

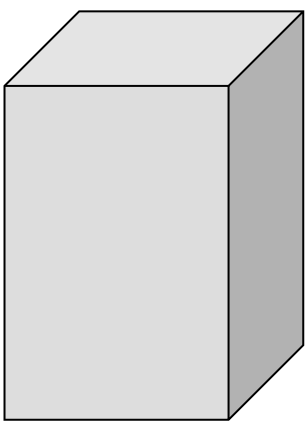
- ➔ Sometimes we will want to maximize volume for a given surface area
- ➔ Sometimes we will want to minimize surface area for a given volume

**Surface Area = 2LW + 2LH + 2WH = 2(LW + LH +WH)**

**Volume = LWH**

## Example #1:

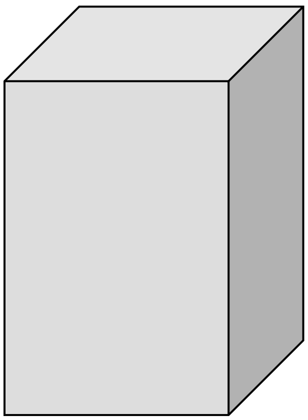
432 $cm^2$ of material is available to make a box with a square base and an open top. Find the largest possible volume of the box. What dimensions maximize the volume of the box?



# ***Ch. 6: Derivative Applications***

**Example #2:**

You want to design a 500 $m^3$, squared based, open top, rectangular steel holding tank. Find the dimensions of the tank that will use the least amount of material.



***Assignment:*** ***Text pg. 279, #23, 24***

***Handout:*** ***#5, 6, 7***

# ***Ch. 6: Derivative Applications***

***Lesson #5: Optimization: Poster Problems / Volume Problems (Sec. 6.3 – day 3)***

***Learning targets:***

i) Using derivatives to solve optimization problems involving poster with margins.
ii) Using derivatives to solve volume optimization problems.

**Example #1:**

The top and bottom margins of a poster are 6 cm and the side margins are 4 cm. If the printed area on the poster is 384 $cm^2$, find the dimensions of the poster with the smallest area.

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**Example #2:**

A box is to be made from a square piece of cardboard by cutting a square out of each corner and turning up the sides to form walls. Given that the cardboard is 40 cm by 40 cm, find the dimensions of the box that will maximize the volume.

***Assignment:*** ***Text pg. 279, #20, 21, 22***

***Handout:*** ***#8***

# ***Ch. 6: Derivative Applications***

***Lesson #6: Related Rates – Circles and Spheres (Sec. 6.4/6.5 – day 1)***

***Learning targets:***

| | |
|---|---|
| i) | Apply implicit differentiation to solve related rates problems involving circles. |
| ii) | Apply implicit differentiation to solve related rates problems involving spheres. |

Implicit differentiation is a form of the **Chain Rule**.

We will once again employ this technique in related rates problems are we find **rates of change** of two or more related variables that are changing **with respect to time**.

$$\frac{d(?)}{dt} \rightarrow rate\ of\ change\ of\ (?)\ with\ respect\ to\ time$$

(?) will be a quantity such as diameter, area, etc.

**Example #1:**

A pebble is dropped into a calm pond causing ripples to form in the shape of concentric circles. The radius of the outside ripple is increasing at a rate of 1.5 m/s.  When the radius is exactly 4 metres, at what rate is the area of the circle formed by the outside ripple changing?

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**Example #2:**

A spherical snowball is melting in such a way that its volume is decreasing at a rate of 1 $cm^3$/min. At what rate is the radius decreasing when the radius is 5 cm?

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**Example #3:**

A spherical balloon increases in diameter at a rate of 10 cm/min. Find the rate of increase of the surface area of the sphere at the instant the surface area is $4\pi\ m^2$.

***Assignment: Text pg. 290, #1, 3, 7, 8. 10***

# ***Ch. 6: Derivative Applications***

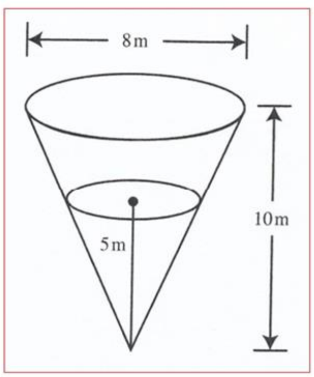
***Lesson #7: Related Rates – Cones (Sec. 6.4/6.5 – day 2)***

***Learning targets:***

| | |
|---|---|
| i) | Apply implicit differentiation to solve related rates problems involving cones. |

**Example #1:**

A water tank is in the shape of an inverted cone. The height of the cone is 10 metres and the diameter of the base is 8 metres as shown in the diagram below. Water is being pumped into the tank at a rate of 2 $m^3$/min. How fast is the water level rising when the water is 5 metres deep?



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**Example #2:**

The nightmare has come to pass. All of Kelley's extensive surgeries and nasal passage scrapings have (unfortunately) gone awry, and he waits in the ear, nose, and throat doctor's office waiting area sewing bloody nose drippings into a conical paper cup at a rate of 2.5 $in^3$/min. The cup is being held with the vertex down and has a height of 4 inches and a base of 3 inches. How fast is the mucous level rising in the cup when the "liquid" is 2 inches deep?

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**Example #3:**

Delores was using a straw to drink a milkshake from a conical container. The radius of the container is 6 cm and its height is 24 cm. If she consumes the milkshake at a rate of 20 $cm^3$/s, at what rate is the height of the milkshake falling when the height is 8 cm?

***Assignment:*** ***Text pg., #290, #11, 12, 13, 14***

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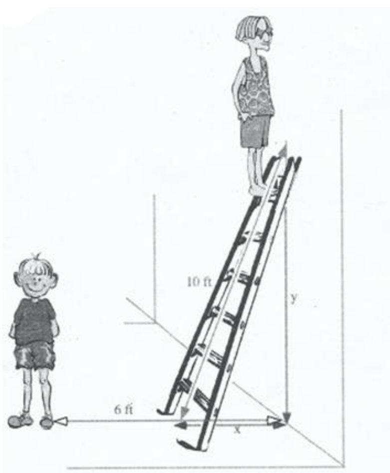
***Lesson #7: Related Rates – Pythagorean Relationship (Sec. 6.4/6.5 – day 3)***

***Learning targets:***

i) Apply implicit differentiation to solve related rates problems the Pythagorean relationship in right triangles.

### Example #1:

Joey is perched precariously at the top of a 10-foot ladder leaning against the back wall of an apartment building when it starts to slide down the wall at a rate of 4 ft per minute. Joey's friend, Lou, is standing on the ground 6 feet away from the wall. How fast is the base of the ladder moving when it hits Lou?



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**Example #2:**

A plane flies horizontally with a speed of 600 km/h at an altitude of 10 km and passes directly over LeBoldus. Find the rate at which the distance from the plane to the school is changing when the horizontal distance of the plane is 20 km from the school (assume the altitude of the plane is not changing).

# ***Ch. 6: Derivative Applications***

**Example #3:**

A car approaches an intersection from the east at a speed of 12 m/s while a truck approaches from the north at a rate of 15 m/s. How fast is the distance between them decreasing when the car is 30 m east of the intersection and the truck is 40 m north of the intersection?

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**Example #4:**

Two cars leave the same intersection at the same time, one traveling north and the other traveling east. The northbound car is traveling at a speed of 90 km/h, while the eastbound vehicle is traveling 60 km/h. How fast is the distance between them increasing 40 min after they have left the intersection?

***Assignment:*** ***Text pg. 284 - 285, #1, 2, 5, 8, 9, 10, 11, 12***

*Calculus 30*

# ***Ch. 6: Derivative Applications***

***Unit Review: Derivative Applications***

***What you need to know and be able to do***

| | |
|---|---|
| i) | ction. |

***Assignment:*** ***Text pg., #***