Sec. 7.4
Reciprocal Functions

Learning Targets:
1) Equations of reciprocal functions
2) Graphs of reciprocal functions
3) Superimposing reciprocals
4) Determining the equations of asymptotes

In general, if y = f(x) is a linear or quadratic function, then $y = \frac{1}{f(x)}$ is the ***reciprocal*** function.

Example:
Linear: $y = x - 3$
Linear Reciprocal: $y = \frac{1}{x-3}$
Quadratic: $y = x^2 - 4$
Quadratic Reciprocal: $y = \frac{1}{x^2-4}$

If you look at a function, point by point, you can determine the points for the reciprocal function by simply taking the reciprocal of the y-coordinate (and keep the same x-coordinate).

If (x, y) is on f(x), then **$(x, \frac{1}{y})$** is on the **reciprocal**.

Example:
f(x) contains the points (**3**, **6**) and (**-2**, **-2**)
The reciprocal contains the points (**3**, $\frac{1}{6}$) and (**-2**, $\frac{-1}{2}$)

Asymptotes:

If f(x) contains a point on the x-axis, its y-coordinate is zero. Since you can't take the reciprocal of zero (*i.e.* $\frac{1}{0}$ *is undefined*), this results in something different...something called a vertical **ASYMPTOTE**.

Vertical Asymptote Rule:
If f(x) has an x-intercept at x=a, then the graph of the reciprocal of f(x) will have a vertical asymptote whose equation is x=a.

Horizontal Asymptote Rule:
In order for a rational number to have a value of zero, its numerator must be zero.

Since every reciprocal function has a numerator of "1", reciprocal functions will never take on a value of zero.

Therefore, reciprocal functions of the form
will never have x-intercepts.

Instead, they will have a **horizontal asymptote** that runs along the x-axis, the equation of which is **y=0**.

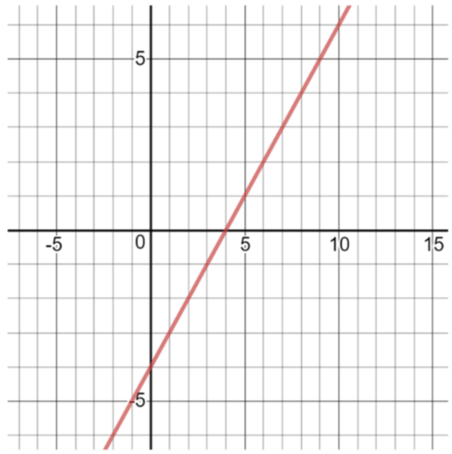
Determining the equations of asymptotes:

Horizontal: always y = 0

Vertical: Set the denominator to zero and solve for x (this is just like finding the NPV for a rational expression)

Comparing a linear function with its reciprocal:

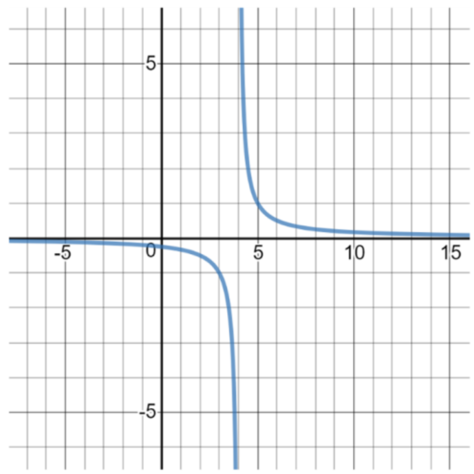
linear function
y = x - 4



has a positive slope
x-intercept at 4
y-intercept at -4

reciprocal function

$$y = \frac{1}{x - 4}$$

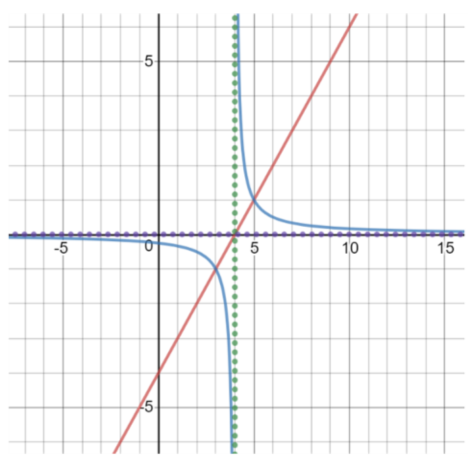


Shape is called a "hyperbola"
(has two separate branches)

no x-intercepts
y-intercept at -1/4
vertical asymptote at x = 4
horizontal asymptote along the x-axis

linear function: y = x - 4

reciprocal function $y = \frac{1}{x - 4}$



vertical asymptote: x = 4

horizontal asymptote: y = 0

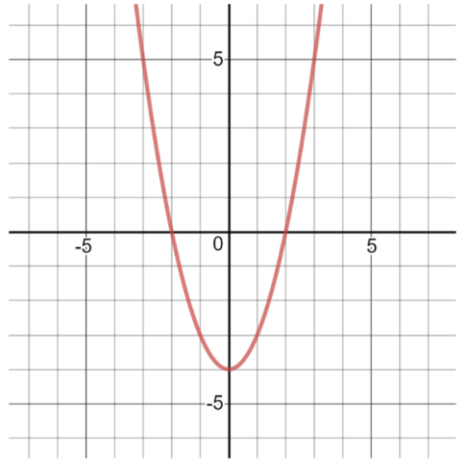
The two graphs intersect at
(3, -1) and (5, 1)

*A linear function and its reciprocal will always intersect at the points where y = 1 and y = -1

Comparing a quadratic function with its reciprocal:

quadratic function
$y = x^2 - 4$

opens up
x-intercepts at -2 and 2
y-intercept at -4



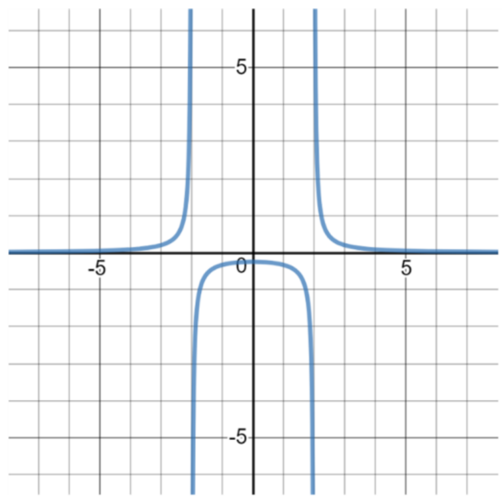
reciprocal function

$$y = \frac{1}{x^2 - 4}$$

no x-intercepts
y-intercept at -1/4

vertical asymptotes at
x = -2 and x = 2

horizontal asymptote along
the x-axis



quadratic function
$y = x^2 - 4$

reciprocal function $y = \frac{1}{x^2 - 4}$

vertical asymptotes: x = -2 and x = 2

horizontal asymptote: y = 0

The two graphs intersect at four points:
- two of the points have a y-coordinate of -1
- the other two points have a y-coordinate of 1.

*A quadratic function and its reciprocal will always intersect at the points where y = 1 and y = -1