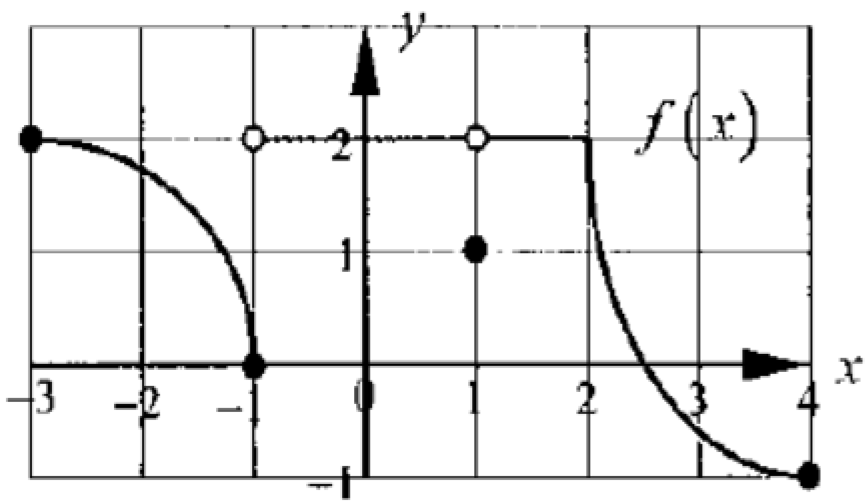
Chapter 3: Limits and Continuity

Key Ideas: ***limits from graphs, direct substitution, limits at infinity, strategies for when direct substitution yields $\frac{0}{0}$, one-sided limits for piecewise functions and rational functions, types of discontinuities, test for continuity at a point***

1. Refer to the following graph and state the value of each quantity, if it exists:



a. $f(1)$ b. $f(-1)$ c. $\lim_{x\to 2} f(x)$ d. $\lim_{x\to 0} f(x)$

e. $\lim_{x\to 1} f(x)$ f. $\lim_{x\to -1^+} f(x)$ g. $\lim_{x\to -1^-} f(x)$ h. $\lim_{x\to -1} f(x)$

2. Determine each of the following limits:

a. $\lim_{x\to 2} 2x^2-4$ b. $\lim_{x\to 3}\frac{x^2-9}{x-3}$ c. $\lim_{x\to -2}\frac{x^3+8}{x+2}$

d. $\lim_{x\to 2^+}\frac{x}{x^2-4}$ e. $\lim_{x\to 9}\frac{\sqrt{x}-3}{x-9}$ f. $\lim_{x\to -5^+}\frac{x+2}{(x+5)^3}$

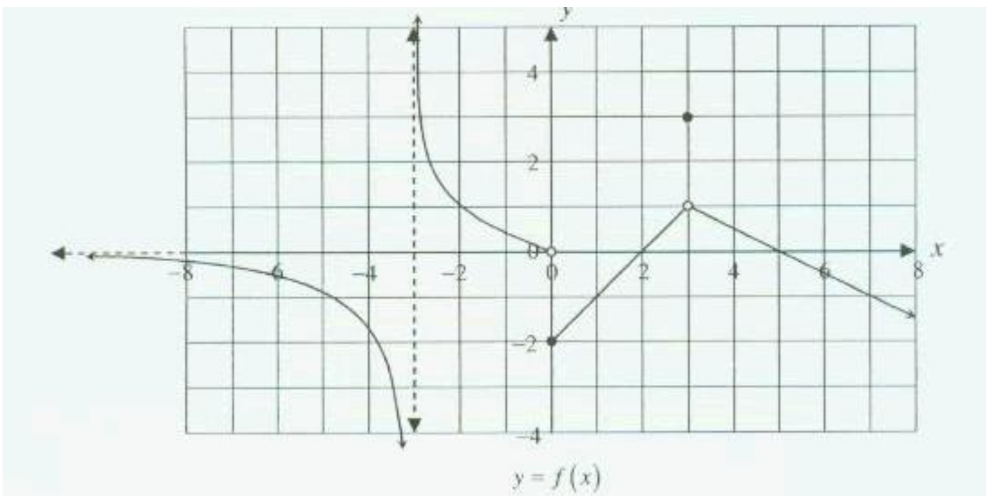
g. $\lim_{x\to\infty}\frac{\sqrt{x^2+x-2}}{x+1}$ h. $\lim_{y\to 0}\frac{\sqrt{y+2}-\sqrt{2}}{-2y}$ i. $\lim_{x\to 10}\frac{\frac{2}{x-4}-\frac{1}{3}}{x-10}$

3. If $f(x) = \begin{Bmatrix} x^3 - 2x^2, x \geq 1 \\ \frac{x+2}{x-2}, \quad x < 1 \end{Bmatrix}$, find:

a. $\lim_{x \to 4} f(x)$ b. $\lim_{x \to -4} f(x)$ c. $\lim_{x \to 2} f(x)$

d. $\lim_{x \to 1^+} f(x)$ e. $\lim_{x \to 1^-} f(x)$ f. $\lim_{x \to 1} f(x)$

4. Use the following graph to determine if the function is continuous at each of the following points. If it is not continuous at the point, state the type of discontinuity.

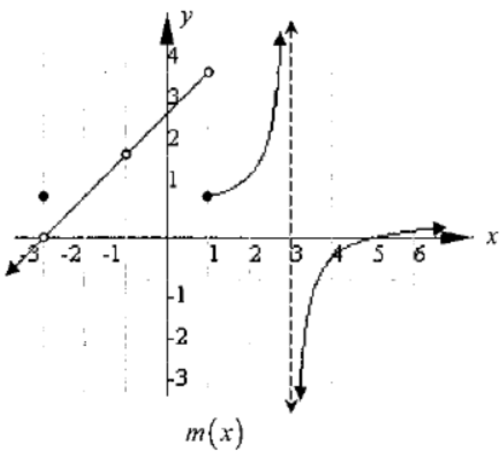


a. x = -2 b. x = -1 c. x = 0

d. x = 2 e. x = -3 f. x = 3 g. x = 5

5. By examining the graph of $m(x)$ below, explain why the function is discontinuous at each of the following by stating which condition of the test for continuity at a point is not satisfied.

a. $x = 3$ b. $x = 1$ c. $x = -1$ d. $x = -3$



6. State the value of k if the function $f(x) = \begin{cases} x^2 + x, & x < -4 \\ kx + 10, & x \geq -4 \end{cases}$ is to be continuous.

7. State if the following functions are continuous or not. If the function is discontinuous state the location of the discontinuity and the type of discontinuity.

   a. $f(x) = \dfrac{x+5}{x-5}$ b. $f(x) = \dfrac{x^2-36}{x+6}$ c. $f(x) = \begin{cases} -2x, x < 0 \\ -x^2, 0 \leq x < 2 \\ 1-2x, x \geq 2 \end{cases}$

Answer Key:

1. a. 1 b. 0 c. 2 d. 2 e. 2
   f. 2 g. 0 h. does not exist

2 a. 4 b. 6 c. 12 d. ∞ e. $\frac{1}{6}$ f. $-\infty$
   g. 1 h. $-\frac{\sqrt{2}}{8}$ i. $-\frac{1}{18}$

3. a. 32 b. $\frac{1}{3}$ c. 0 d. -1 e. -3 f. dne

4. a. continuous
   b. continuous
   c. not continuous – jump discontinuity
   d. continuous
   e. not continuous – infinite discontinuity
   f. not continuous – removable discontinuity
   g. continuous

5. a. Condition #1 : $m(3)$ does not exist (vertical asymptote)
   b. Condition #2: $\lim\limits_{m \to 1} m(x)$ does not exist (jump in the graph)
   c. Condition #1: $m(-1)$ does not exist (hole in the graph)
   d. Condition #3: $\lim\limits_{m \to -3} m(x) \neq m(-3)$

6. $k = -\frac{1}{2}$

7. a. discontinuous at x = 5, infinite discontinuity
   b. discontinuous at x = -6, removable discontinuity
   c. discontinuous at x = 2, jump discontinuity (continuous at x = 0)