Chapter 1: Background Essentials

Key Ideas: ***interval notation, factoring, zero and division, inequalities, absolute value***

1. Write each of the following inequalities using interval notation
a. $\{x: x \leq 5\}$ b. $\{x: -7 \leq x < 8\}$ c. $\{x: x \geq 1 \text{ or } x < -6\}$

2. Write the following intervals using inequalities
a. $[-3, \infty)$ b. $(-3, 7]$

3. Use interval notation to describe the results of the following:
a. $[-3, 5) \cup (1, 8)$ b. $[-1, 6) \cap (1, 7)$

4. Factor the following over the set of rational numbers
a. $6x^2 - 11x + 4$
b. $5x^3 - 20x$
c. $a^3 - 216$
d. $x^2 - 5x + 6$
e. $64x^3 + 125$
f. $(3x + 2)^2 - (x + 3)^2$
g. $\sin^3 x - \sin x$
h. $x^4 - 81$

5. Factor the following as a difference of squares over the set of real numbers.
a. $3x^2 - 24$ b. $y - 20$

6. Factor the following as a sum or difference of cubes over the set of real numbers
a. $8a^3 + 5$ b. $d - 32$

7. Factor $64x^6 - 1$ over the set of rationals by:
a. initially regarding the expression as a difference of squares
b. initially regarding the expression as a difference of cubes

8. Determine which of the following trinomials can be factored over the set of rational.
a. $2x^2 - 5x - 25$ b. $3x^2 - x + 4$ c. $x^2 - 7x + 8$

9. Factor by eliminating all fractional coefficients.
a. $x^2 + \frac{7}{4}x - \frac{1}{2}$ b. $\frac{1}{3}x^3 - \frac{3}{4}xy^2$

10. Factor each of the following expressions so that the second factor contains no negative exponents or fractional coefficients.
a. $35x^{-3}y^2 - 20x^2y^{-4}$ b. $\frac{5}{6}x^{-3/2}y^{2/5} - \frac{3}{4}x^{5/2}y^{-8/5}$

11. Find the value(s) of x, if any, for which each function is (i) zero, (ii) undefined, (iii) indeterminate
a. $f(x) = \dfrac{2x^3 - 32x}{x^2 + x - 12}$ b. $f(x) = \dfrac{2x^2 + 9x + 4}{4x^3 - 12x^2 - x + 3} = \dfrac{(2x+1)(x+4)}{(2x+1)(2x-1)(x-3)}$

12. Rewrite the following absolute value expressions without absolute value signs

a. $|18+3x-x^2|$ b. $\left|\frac{x^2-25}{2-x}\right|$

13. Solve the following equation $|3x-7|=20$

14. Solve the following inequalities:

a. $|2x+3| \leq 15$ b. $\left|\frac{x}{x+4}\right| > \frac{2}{3}$

ANSWER KEY

1a. $(-\infty, 5]$ b. $[-7, 8)$ c. $(-\infty, -6) \cup [1, \infty)$

2a. $x \geq -3$ b. $-3 < x \leq 7$

3a. $[-3, 8)$ b. $(1, 6)$

4a. $(2x-1)(3x-4)$ b. $5x(x-2)(x+2)$ c. $(a-6)(a^2+6a+36)$
d. $(x-2)(x-3)$ e. $(4x+5)(16x^2-20x+25)$ f. $(4x+5)(2x-1)$
g. $\sin x(\sin x-1)(\sin x+1)$ h. $(x-3)(x+3)(x^2+9)$

5a. $3(x-2\sqrt{2})(x+2\sqrt{2})$ b. $(\sqrt{y}-2\sqrt{5})(\sqrt{y}+2\sqrt{5})$

6a. $(2a+\sqrt[3]{5})(4a^2-2a\sqrt[3]{5}+\sqrt[3]{25})$ b. $(\sqrt[3]{d}-2\sqrt[3]{4})(\sqrt[3]{d^2}+2\sqrt[3]{4d}+8\sqrt[3]{2})$

7a. $(2x-1)(2x+1)(4x^2+2x+1)(4x^2-2x+1)$
7b. $(2x-1)(2x+1)(16x^4+4x^2+1)$

8a. $D = 225$ (yes) b. $D = -47$ (no) c. $D = 17$ (no)

9a. $\frac{1}{4}(4x-1)(x+2)$ b. $\frac{1}{12}x(2x-3y)(2x+3y)$

10a. $5x^{-3}y^{-4}(7y^6-4x^5)$ b. $\frac{1}{12}x^{-\frac{3}{2}}y^{-\frac{8}{5}}(10y^2-9x^4)$

11a i. $x = 0$ and 4 ii. $x = 3$ iii. $x = -4$
11b. i. $x = -4$ ii. $x = ½$ and 3 iii. $x = -1/2$

12a. $|18+3x-x^2| = \begin{cases} 18+3x-x^2, \text{ if } x \in [-3,6] \\ -(18+3x-x^2), \text{ if } x \in (-\infty,-3)\cup(6,\infty) \end{cases}$

b.

$\frac{x^2-25}{2-x} = \frac{(x+5)(x-5)}{2-x}$	Test values on a number line to determine positive and negative areas $---------0+++++++++++++++++++++++ \ (x+5)$ $--------------------0++++++++++++ \ (x-5)$ $+++++++++++++++0--------------- \ (2-x)$ $-5 \quad 2 \quad 5$ $+++++++0-------\varnothing+++------------- \ \frac{(x^2-25)}{(2-x)}$ There is no value at 2 as it is a vertical asymptote.	$\left\lvert\frac{x^2-25}{2-x}\right\rvert = \begin{cases} \frac{x^2-25}{2-x}, \text{ if } x \in (-\infty,-5]\cup(2,5] \\ -\left(\frac{x^2-25}{2-x}\right), \text{ if } x \in (-5,2)\cup(5,\infty) \end{cases}$

13. $x = 9$, $x = -13/3$

14. a. [-9, 6] b. $\left(-\infty,-\frac{8}{5}\right)\cup(8,\infty)$