Example 2

Use the laws of logarithms to simplify and evaluate each expression.

a) $\log_3 9\sqrt{3}$

b) $\log_5 1000 - \log_5 4 - \log_5 2$

c) $2\log_3 6 - \dfrac{1}{2}\log_3 64 + \log_3 2$

a) $\log_3 9\sqrt{3} = \log_3 9 + \log_3 \sqrt{3}$

$= \log_3 3^2 + \log_3 3^{1/2}$

$= 2 + \frac{1}{2}$

$= \frac{5}{2}$

b) $\log_5 1000 - \log_5 4 - \log_5 2$

$= \log_5 \dfrac{1000}{4 \cdot 2}$

$= \log_5 125$

$= \log_5 5^3$

$= 3$

c) $2\log_3 6 - \frac{1}{2}\log_3 64 + \log_3 2$

$= \log_3 6^2 - \log_3 64^{1/2} + \log_3 2$

$= \log_3 36 - \log_3 8 + \log_3 2$

$= \log_3 \frac{36 \cdot 2 \left(\frac{72}{8}\right)}{8}$

$= \log_3 9$

$= \log_3 3^2 = 2$

Example 3

Write each expression as a single logarithm in simplest form. State the restrictions on the variable.

a) $4 \log_3 x - \frac{1}{2}(\log_3 x + 5 \log_3 x)$

b) $\log_2 (x^2 - 9) - \log_2 (x^2 - x - 6)$

a) $4 \log_3 x - \frac{1}{2}\left(\log_3 x + 5 \log_3 x\right)$

$= \log_3 x^4 - \frac{1}{2}\left(\log_3 x + \log_3 x^5\right)$

$= \log_3 x^4 - \frac{1}{2}\left(\log_3 x^6\right)$

$= \log_3 x^4 - \log_3 (x^6)^{\frac{1}{2}}$

$$= \log_3 x^4 - \log_3 x^3$$

$$= \log_3 \frac{x^4}{x^3}$$

$$= \log_3 x \qquad x > 0$$

b) $\log_2(x^2-9) - \log_2(x^2-x-6)$

$= \log_2 \dfrac{x^2-9}{x^2-x-6}$

$= \log_2 \dfrac{(x+3)(x-3)}{(x+2)(x-3)}$

$= \log_2 \dfrac{x+3}{x+2}$

$\left.\begin{array}{l} x \neq -2 \\ x+3 > 0 \\ x > -3 \end{array}\right\} x > -2$

Key Ideas

- Let P be any real number, and M, N, and c be positive real numbers with $c \neq 1$. Then, the following laws of logarithms are valid.

Name	Law	Description
Product	$\log_c MN = \log_c M + \log_c N$	The logarithm of a product of numbers is the sum of the logarithms of the numbers.
Quotient	$\log_c \dfrac{M}{N} = \log_c M - \log_c N$	The logarithm of a quotient of numbers is the difference of the logarithms of the dividend and divisor.
Power	$\log_c M^P = P \log_c M$	The logarithm of a power of a number is the exponent times the logarithm of the number.

- Many quantities in science are measured using a logarithmic scale. Two commonly used logarithmic scales are the decibel scale and the pH scale.

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