Base Graph

$y = \log_2 x$

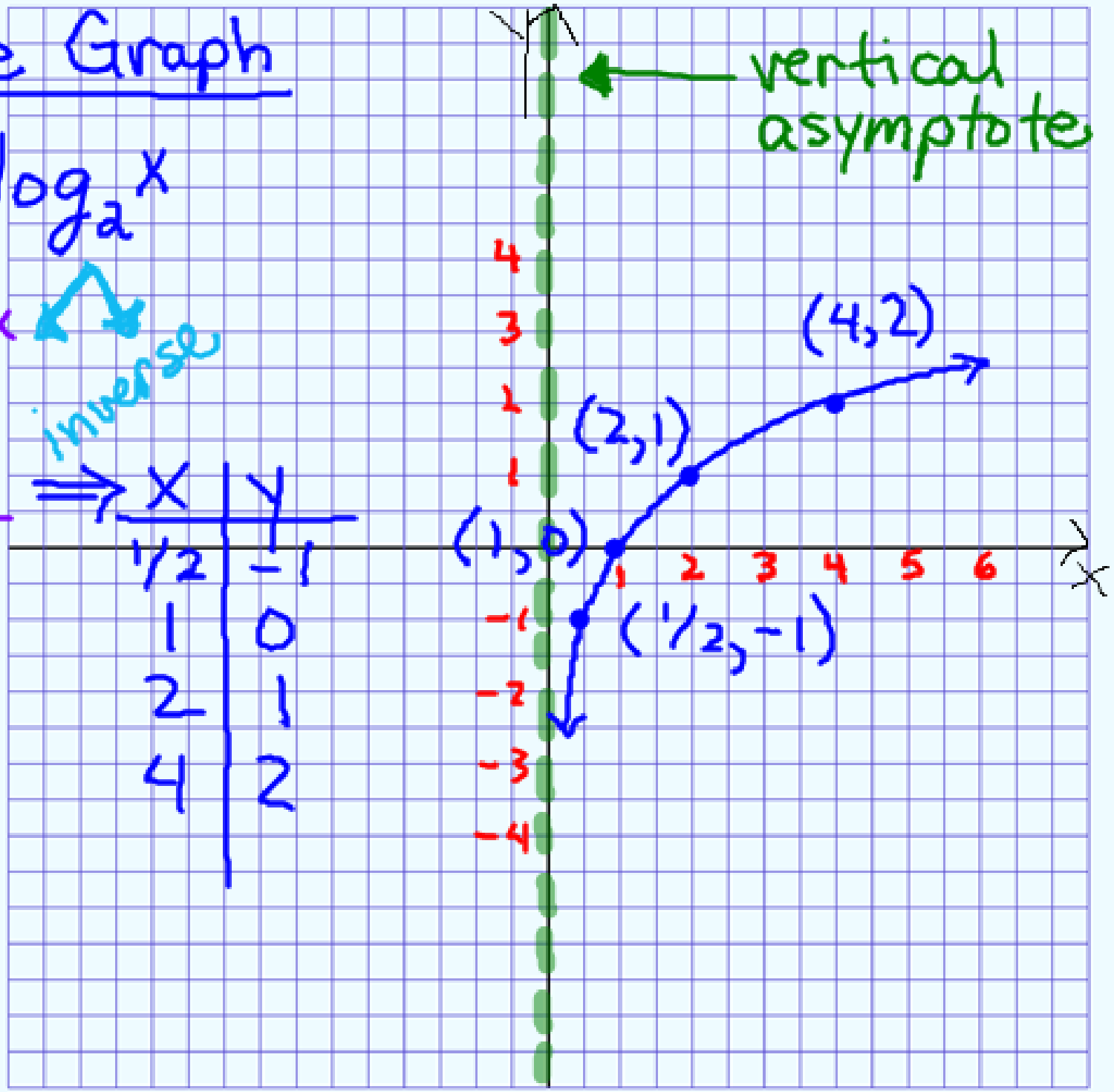
$y = 2^x$

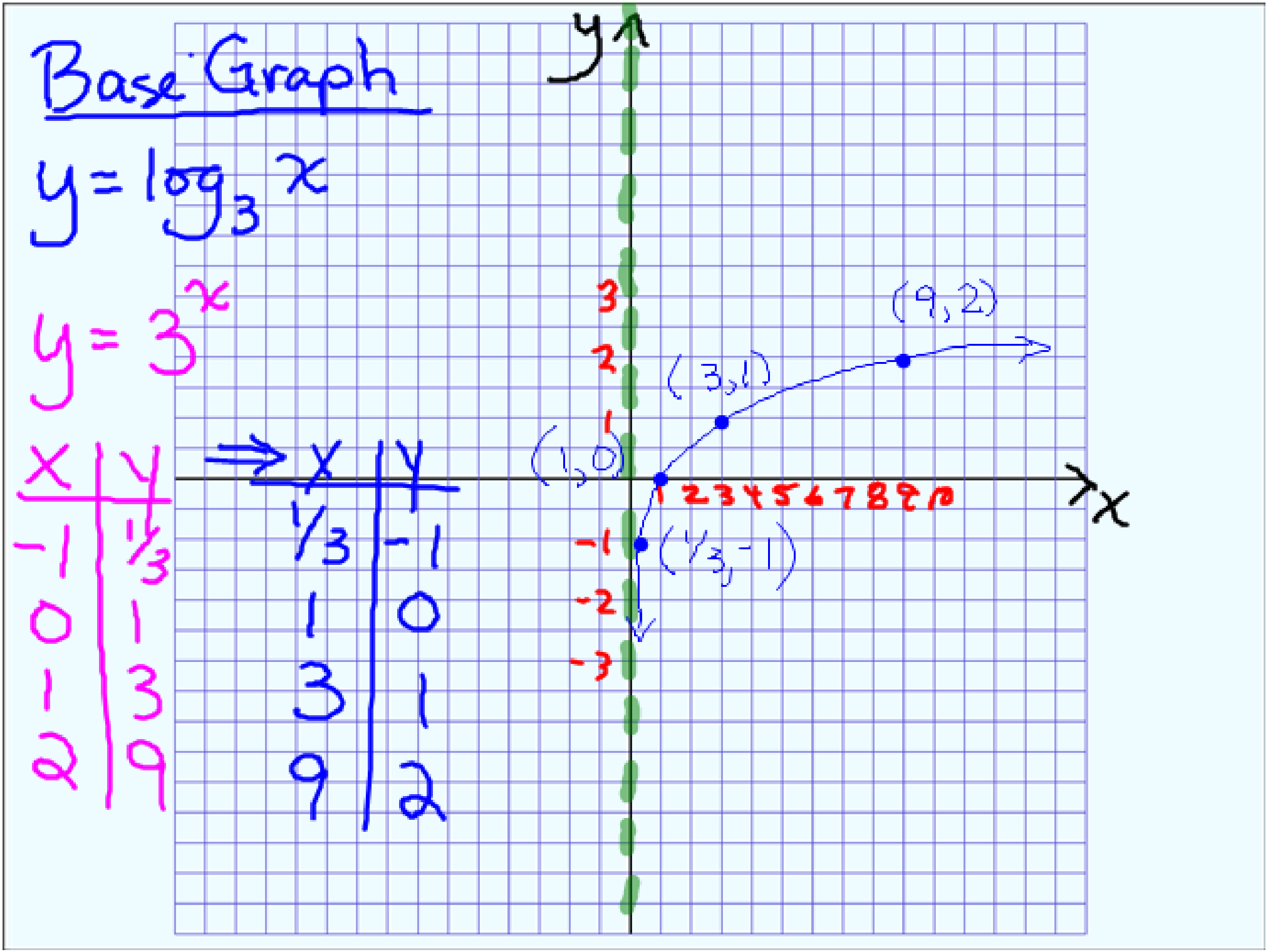
inverse

x	y
-1	1/2
0	1
1	2
2	4

$\Rightarrow$

x	y
1/2	-1
1	0
2	1
4	2




Base Graph
$y = \log_3 x$
$y = 3^x$
x | y
-1 | 1/3
0 | 1
1 | 3
2 | 9
⇒
x | y
1/3 | -1
1 | 0
3 | 1
9 | 2
y
x
3
2
1
-1
-2
-3
1 2 3 4 5 6 7 8 9 10
(9, 2)
(3, 1)
(1, 0)
(1/3, -1)

8.2 Transformations of Logarithmic Functions

The graph of the logarithmic function $y = a \log_c (b(x - h)) + k$ can be obtained by transforming the graph of $y = \log_c x$. The table below uses mapping notation to show how each parameter affects the point (x, y) on the graph of $y = \log_c x$.

Parameter	Transformation
a	$(x, y) \rightarrow (x, ay)$
b	$(x, y) \rightarrow \left(\frac{x}{b}, y\right)$
h	$(x, y) \rightarrow (x + h, y)$
k	$(x, y) \rightarrow (x, y + k)$

These transformations are no different than all the other transformations we have done this year.

$y = \log_c x \Rightarrow y = a \log_c (b(x-h)) + k$

$(x, y) \Rightarrow \left(\frac{x}{b} + h, ay + k\right)$

Find each point under the transformation:

$y = -3\log_2 4(x-6)+2$

$a = -3$

$b = 4$

$h = 6$

$k = 2$

(4, 2) -> $\left(\frac{4}{4}+6,\ -3(2)+2\right)$

$= (7, -4)$

(64, 6) -> $\left(\frac{64}{4}+6,\ -3(6)+2\right)$

$= (22, -16)$

Example 1

a) Use transformations to sketch the graph of the function $y = \log_3 (x + 9) + 2$.

b) Identify the following characteristics of the graph of the function.

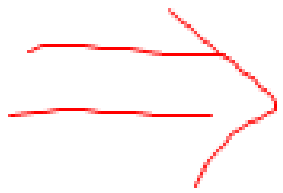
i) the equation of the asymptote **ii)** the domain and range

iii) the y-intercept, if it exists **iv)** the x-intercept, if it exists

Start by sketching $y = log_3 x$

$h = -9$ $k = +2$

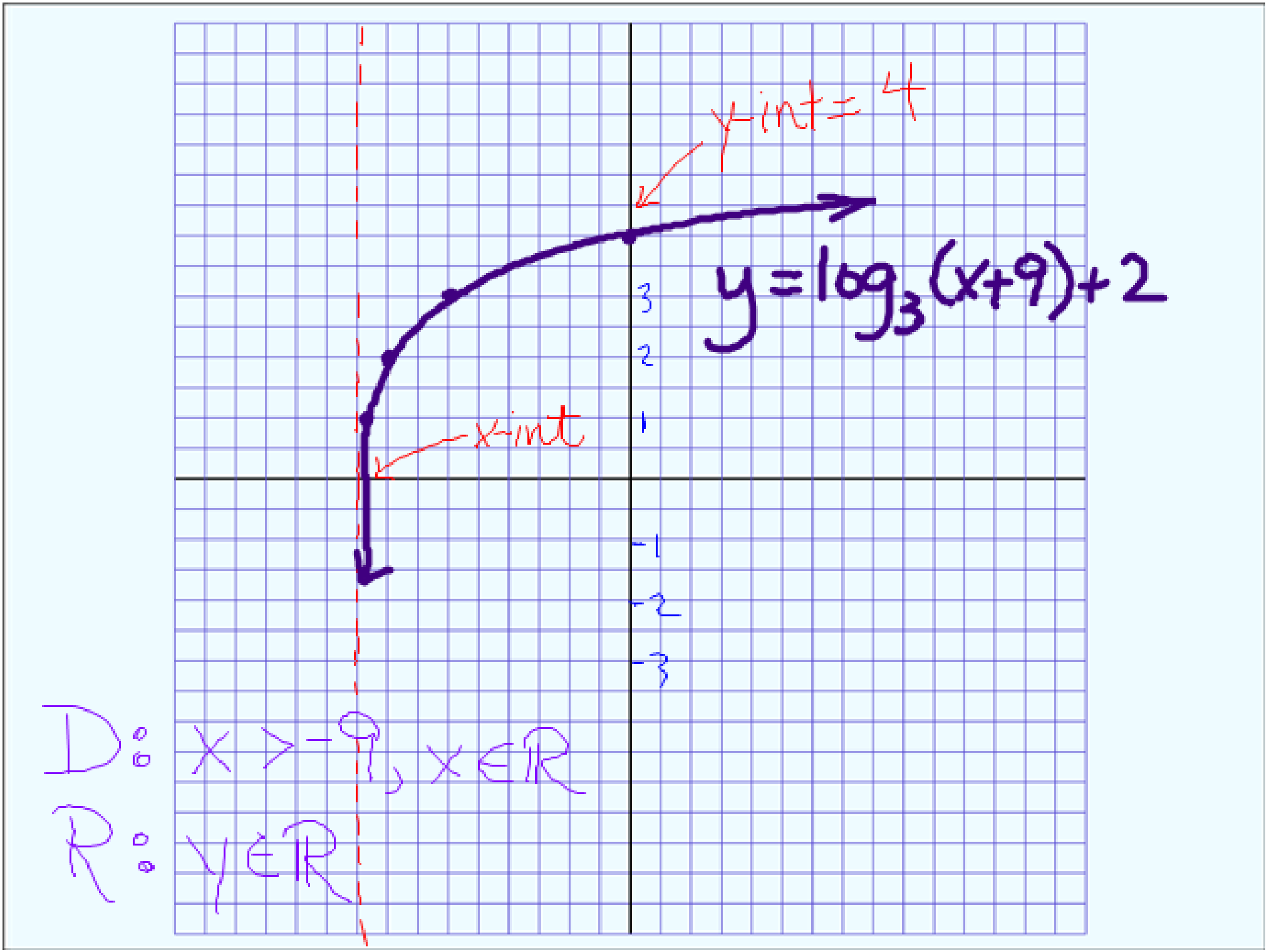
x	y
1/3	−1
1	0
3	1
9	2



x	y
$-8\frac{2}{3}$	1
−8	2
−6	3
0	4

equation of asymptote

$x = -9$


y-int = 4
$y = \log_3(x+9)+2$
x-int
3
2
1
-1
-2
-3
D: $x > -9,\ x \in \mathbb{R}$
R: $y \in \mathbb{R}$

Your Turn

a) Use transformations to sketch the graph of the function $y = 2\log_3(-x + 1)$.

b) Identify the following characteristics.

i) the equation of the asymptote

ii) the domain and range

iii) the y-intercept, if it exists

iv) the x-intercept, if it exists

Start by sketching $y = log_3 x$

$y = 2\log_3(-x+1)$

$y = 2\log_3(-1(x-1))$

$a = 2$

$b = -1$

$h = 1$

$k = 0$

VA at $x = 1$

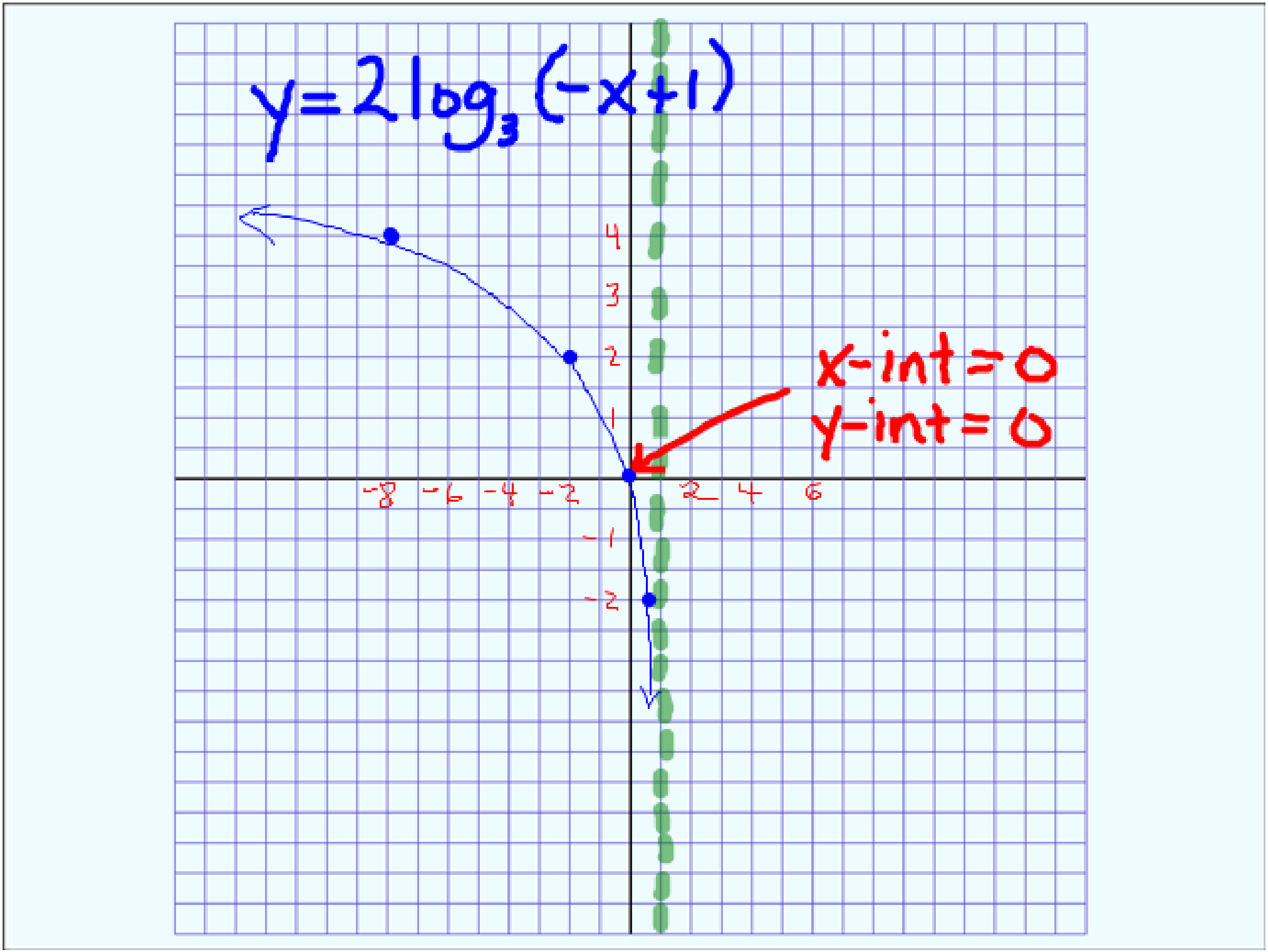
$x > 1, x \in \mathbb{R}$

x	y
$1/3$	-1
1	0
3	1
9	2

$x \in \mathbb{R}$

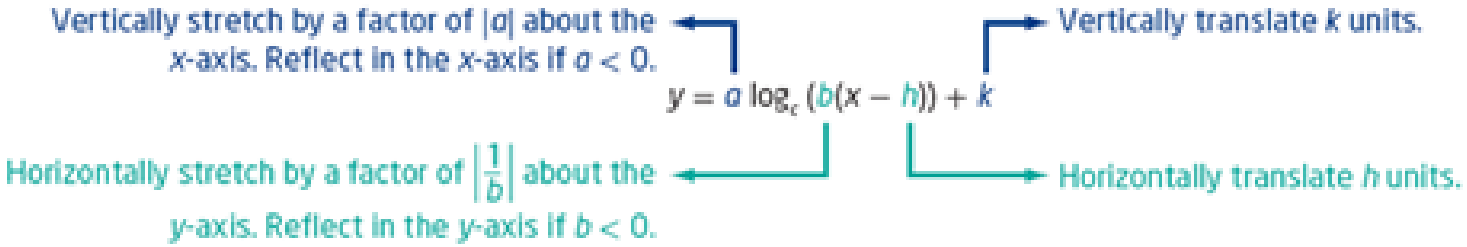
⇒

x	y
$2/3$	-2
0	0
-2	2
-8	4


$y=2\log_3(-x+1)$
x-int = 0
y-int = 0
-8
-6
-4
-2
2
4
6
4
3
2
1
-1
-2

Key Ideas

- To represent real-life situations, you may need to transform the basic logarithmic function $y = \log_b x$ by applying reflections, stretches, and translations. These transformations should be performed in the same manner as those applied to any other function.
- The effects of the parameters a, b, h, and k in $y = a \log_c (b(x - h)) + k$ on the graph of the logarithmic function $y = \log_c x$ are shown below.



- Only parameter h changes the vertical asymptote and the domain. None of the parameters change the range.

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#’s 1,3,5,6,7,9,10,13,14