6.3 Proving Identities

To prove that an identity is true for all permissible values, it is necessary to express both sides of the identity in equivalent forms. One or both sides of the identity must be algebraically manipulated into an equivalent form to match the other side.

You cannot perform operations across the equal sign when proving a potential identity. Simplify the expressions on each side of the identity independently.

Example Prove the following:

$$1 - sin^2x = sinxcosxcotx$$

$\cos^2 x$

$\sin x \cos x \frac{\cos x}{\sin x}$

$\cos^2 x$

QED

$\sin^2 x + \cos^2 x = 1$

$\cos^2 x = 1 - \sin^2 x$

$\cot x = \frac{\cos x}{\sin x}$

Example: Prove the following: $\frac{tanxcosx}{cscx} = 1 - cos^2x$

$\sin^2x + \cos^2x = 1$

$\sin^2x = 1 - \cos^2x$

$\tan x = \frac{\sin x}{\cos x}$

$\csc x = \frac{1}{\sin x}$

$\frac{\frac{\sin x}{\cos x} \cdot \cos x}{\frac{1}{\sin x}}$

$\frac{\sin x}{\cos x} \cdot \cos x \div \frac{1}{\sin x}$

$\frac{\sin x}{\cancel{\cos x}} \cdot \cancel{\cos x} \cdot \sin x$

$\sin^2 x$

$\sin^2 x$

QED

Example Prove the following:

$$\frac{sinx - sinxcos^2x}{sin^2x} = sinx$$

GCF of sinx factored out

$\sin^2x + \cos^2x = 1$

$\sin^2x = 1 - \cos^2x$

$$\frac{\sin x\,(1-\cos^2 x)}{\sin^2 x}$$

$$\frac{\sin x\,(\sin^2 x)}{\sin^2 x}$$

$\sin x$

QED

Example: Prove the following:

$$\frac{1}{sinx-1}+\frac{1}{sinx+1}=\frac{-2tanx}{cosx}$$

$$\frac{(sinx+1)}{(sinx+1)(sinx-1)}\frac{1}{}+\frac{1}{sinx+1}\cdot\frac{(sinx-1)}{(sinx-1)}$$

$$\frac{sinx+1}{sin^2x-1}+\frac{sinx-1}{sin^2x-1}$$

$$\frac{2sinx}{sin^2x-1}$$

$$\frac{2sinx}{-cos^2x}$$

$$-\frac{2sinx}{cos^2x}$$

$$-2\left(\frac{sinx}{cosx}\right)$$

$$\frac{}{cosx}$$

$$-\frac{2sinx}{cosx}\div cosx$$

$$-\frac{2sinx}{cosx}\times\frac{1}{cosx}$$

$$-\frac{2sinx}{cos^2x}$$

QED

Get a CD for the right side:

$$(sinx-1)(sinx+1)$$

$$sin^2x+cos^2x=1$$

$$sin^2x-1=-cos^2x$$

$$tanx=\frac{sinx}{cosx}$$

Assignment:

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