Chapter 5:
Graphical Applications of the Derivative

Lesson #1: Higher Order Derivatives
(Section 5.1)

Learning Targets:

i) Defining higher order derivatives

ii) Determining the first and second derivatives for a variety of explicitly defined functions.

iii) Using implicit differentiation to determine the first and second derivatives when y is implicitly defined in terms of x.

Overview:

In Chapter 5 we will learn how derivatives will help us in drawing the graph of a function with greater accuracy.

The graphing techniques we will be learning rely on the examination of "**higher order derivatives**" for the functions.

Before we begin the graphing applications, we must learn about those higher order derivatives.

Higher Order Derivatives:

If $f(x)$ is a differentiable function, then $f'(x)$ is known as its first derivative.

If $f'(x)$ is differentiable, then its derivative, the **second derivative**, is known as $f''(x)$ (read as "**f double prime of x**")

Other notation $\quad \boldsymbol{f'(x) = \frac{dy}{dx}} \qquad \boldsymbol{f''(x) = \frac{d^2y}{dx^2}}$

Other higher order derivatives, such as the third, fourth, fifth, etc. are commonly denoted as follows:

$$f'''(x)\,,\ \ f^{(4)}(x)\,,\ \ f^{(5)}(x),\ \ etc.$$

$$\frac{d^3y}{dx^3}\,,\ \ \frac{d^4y}{dx^4}\,,\ \ \frac{d^5y}{dx^5}\,,\ \ etc$$

For now, we will focus on determining the second derivative for a variety of functions.

Example1: Find the first two derivatives of the following:

$$f(x) = x^6 + 5x^4 - 3x^3 + x$$

$$f'(x) = 6x^5 + 20x^3 - 9x^2 + 1$$

$$f''(x) = 30x^4 + 60x^2 - 18x$$

$$= 6x(5x^3 + 10x - 3)$$

Example 2: Find $\frac{d^2y}{dx^2}$ for each of the following:

a) $y = \frac{2x+1}{x-1}$

$$y' = \frac{(x-1)(2)-(2x+1)(1)}{(x-1)^2}$$

$$y' = \frac{\cancel{2x}-2-\cancel{2x}-1}{(x-1)^2} = \frac{-3}{(x-1)^2}$$

$$y'' = \frac{\cancel{(x-1)^2(0)} - (-3)(2)(x-1)(1)}{(x-1)^4}$$

$$y'' = \frac{6\cancel{(x-1)}}{(x-1)^{\cancel{4}\,3}}$$

$$y'' = \frac{6}{(x-1)^3}$$

b) $y = x^2\sqrt{x-1}$ $= x^2(x-1)^{1/2}$

$$y' = 2x(x-1)^{1/2} + x^2\left(\tfrac{1}{2}(x-1)^{-1/2}(1)\right)$$

$$y' = 2x(x-1)^{1/2} + \tfrac{1}{2}x^2(x-1)^{-1/2}$$

$$y' = \tfrac{1}{2}x(x-1)^{-1/2}\left[4(x-1) + x\right]$$

$$y' = \frac{x(4x-4+x)}{2(x-1)^{1/2}} = \frac{5x^2-4x}{2(x-1)^{1/2}}$$

$$y'' = \frac{2(x-1)^{1/2}(10x-4) - (5x^2-4x)(x-1)^{-1/2}}{4(x-1)}$$

$$y'' = \frac{(x-1)^{-1/2}\left[\overset{10x^2-14x+4}{2(x-1)(10x-4)} - (5x^2-4x)\right]}{4(x-1)}$$

$$y'' = \frac{15x^2-24x+8}{4(x-1)^{3/2}}$$

c) $x^2 - y^2 = 4$

$$2x - 2y\frac{dy}{dx} = 0$$

$$2x = 2y\frac{dy}{dy}$$

$$\frac{2x}{2y} = \frac{dy}{dx}$$

$$\frac{x}{y} = \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{x}{y}$$

$$-1(x^2 - y^2) = (4)-1$$

$$y^2 - x^2 = -4$$

$$\frac{d^2y}{dx^2} = \frac{y(1) - x\frac{dy}{dx}}{y^2}$$

sub in -4

$$\frac{d^2y}{dx^2} = \frac{y - x\left(\frac{x}{y}\right)}{y^2} = \frac{y - \frac{x^2}{y}}{y^2} \cdot \frac{y}{y} = \frac{y^2 - x^2}{y^3}$$

$$\frac{d^2y}{dx^2} = -\frac{4}{y^3}$$

Assignment

Page 220

Written Exercises: #1 - 11, 13, 14, 17, 20, 21