Ch. 3 : Polynomial Functions

3.1 Characteristics of Polynomial Functions

Polynomial Function- is a function in the form:

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \cdots + a_2 x^2 + a_1 x + a_0$$

where:

n is a whole number → exponents can't be fractions, decimals, or negative.

x- variable

the coefficients are real numbers.

Examples:

$y = 3x + 2$ $\quad$ $y = x^4 + 4$

$y = 2x^2 - x + 3$ $\quad$ $y = x^6 + x^5 + x^4 + x^3 + x^2 + x + 1$

The **degree of a polynomial** is determined by the largest **exponent** of an individual term.

$f(x) = 5x^4 + 3x^2 - 2x$ **degree 4**

Factored Form

$f(x) = (x+2)(x+3)$ **degree 2**

$f(x) = x^2 + 5x + 6$

You try!

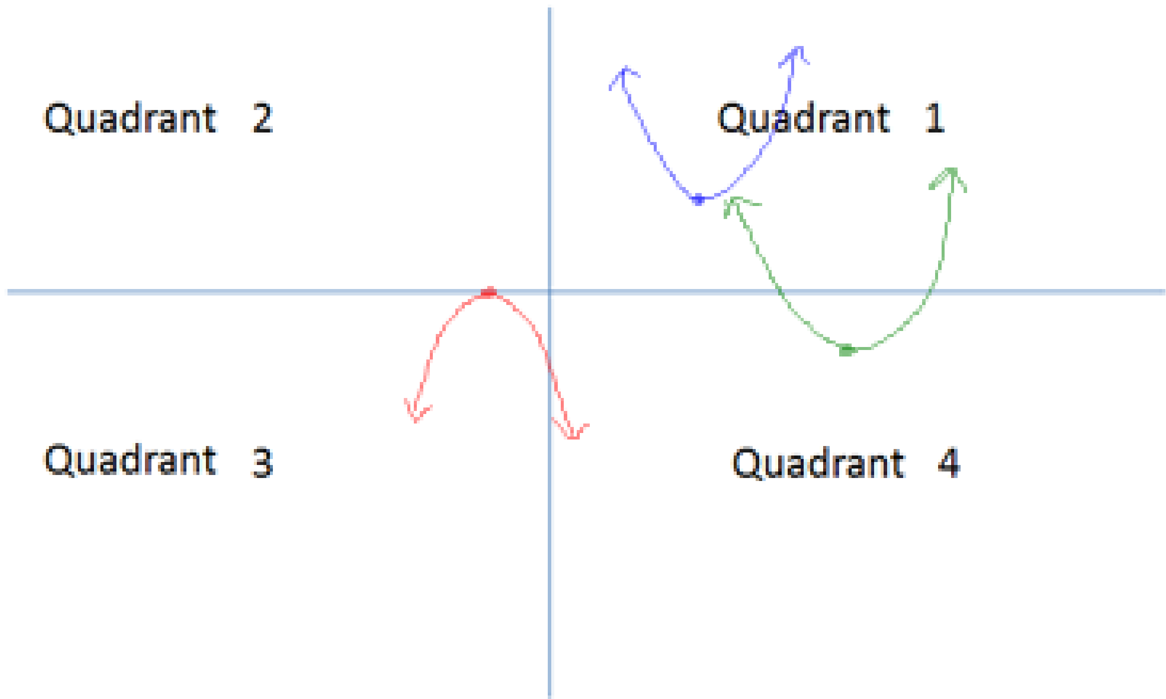
$$f(x) = (x^2+2)(x^3+2x-1)(x-2)$$

2 + 3 + 1 ∴ degree 6

What do you think the degree is?

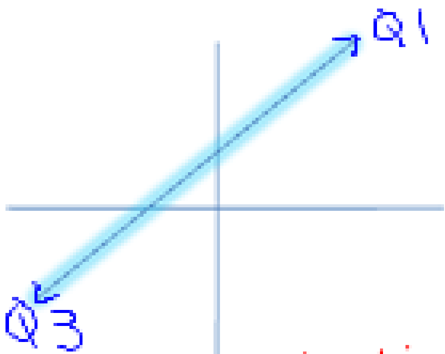
The **degree** also determines the maximum number of **x-intercepts.**

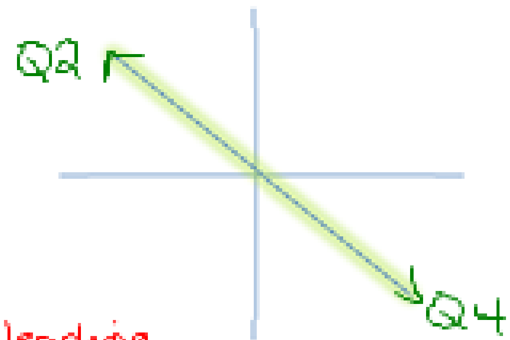
Degree 0- no x intercepts
Degree 1- max one x intercept
Degree 2- max 2 intercepts
Degree 3- max 3 intercepts and so forth.



End Behaviour

Any polynomial with an odd (1,3,5...) degree will have a similar end behavior as a line.

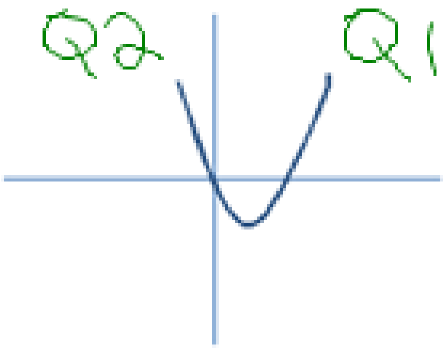




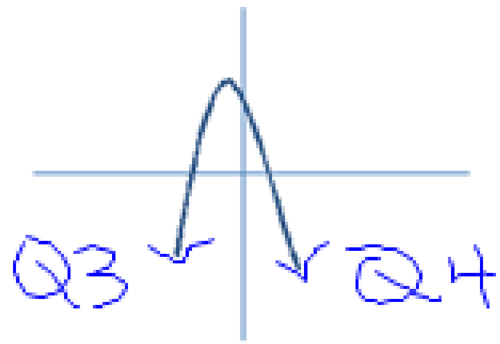
$f(x) = a_n x^n + a_{n-1} x^{n-1} \dots$ ← leading term, a_n = leading coefficient

If the polynomial has an odd degree and a positive leading coefficient, the graph is moving up, starting in quadrant 3 and ending in quadrant 1.	If the polynomial has an odd degree and a negative leading coefficient, the graph is moving down, starting in quadrant 2 and ending in quadrant 4.

Any polynomial with an even degree(2,,4,6,8…) will have the same end behavior as a parabola. Parabola has a degree of 2



If the polynomial has an even degree and a positive leading coefficient, the graph starts in quadrant 2 and ends in quadrant 1.



If the polynomial has an even degree and a negative leading coefficient, the graph starts in quadrant 3 and ends in quadrant 4.

Any polynomial where the leading coefficient is (+) will extend up into quadrant 1. Any polynomial where the leading coefficient is (-) will extend down into quadrant 4.

Identifying Polynomial Functions

State whether each function is a polynomial? If so state the degree, the leading coefficient, and the constant term.

a) $y = 2x^2 - 3x + 2$ yes, degree 2, LC = 2, Const = 2

b) $y = 3^x + 5$ not polynomial

c) $g(x) = (3x + 2)(x - 6)$ yes, degree 2, LC = 3, const = -12

d) $g(x) = x^{-2} + 7x^3$ not polynomial

e) $y = \sin x$ not polynomial

You try!

Which functions are polynomials? Justify your answer. State the degree, the leading coefficient, and the constant term of each polynomial function.

a) $g(x) = \sqrt{x} + 5$

b) $f(x) = 3x^4$

c) $y = |x|$

d) $y = 2x^3 + 3x^2 - 4x - 1$

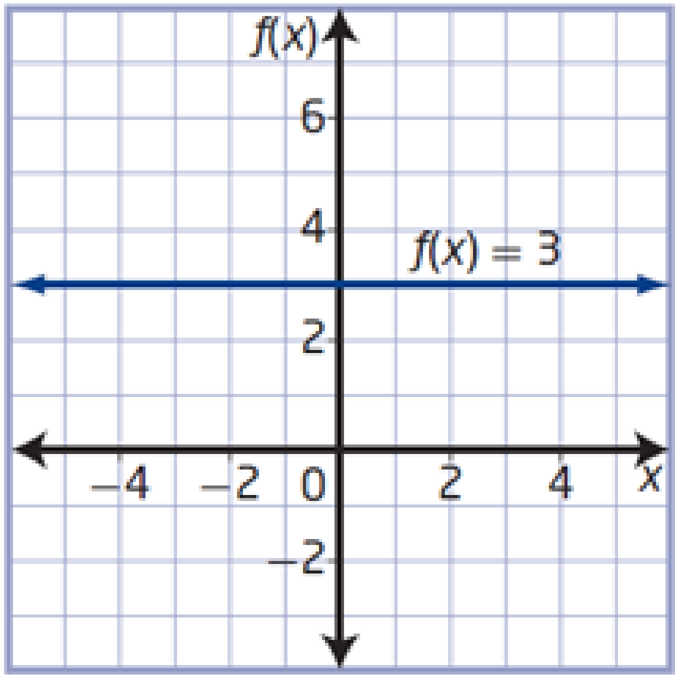
	Yes/No	Degree	Lead coeff	Constant
a)	No	—	—	—
b)	Yes	4	3	0
c)	No	—	—	—
d)	Yes	3	2	-1

See page 109 textbook

Degree 0: Constant Function

Even degree

Number of x-intercepts: 0 (for $f(x) \neq 0$)



Example: $f(x) = 3$

End behaviour: extends horizontally

Domain: $\{x \mid x \in R\}$

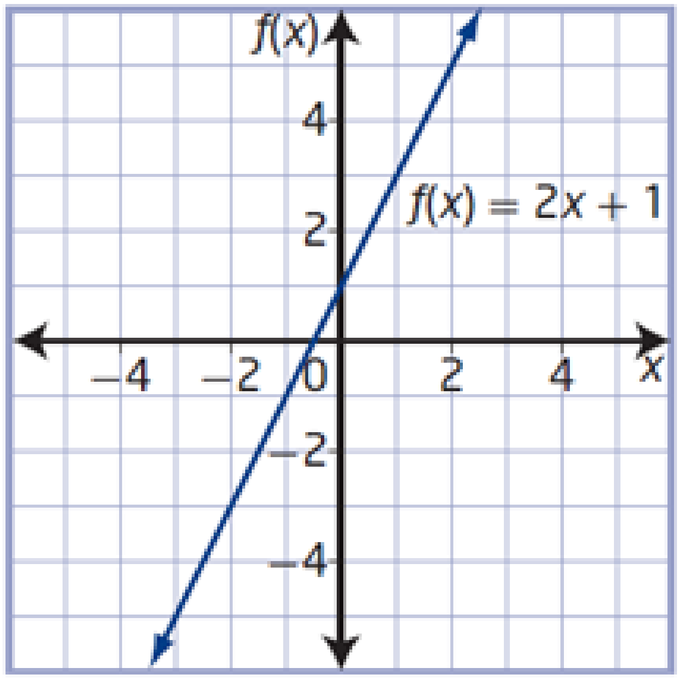
Range: $\{3\}$

Number of x-intercepts: 0

Degree 1: Linear Function

Odd degree

Number of *x*-intercepts: 1



Example: $f(x) = 2x + 1$

End behaviour: line extends down into quadrant III and up into quadrant I

Domain: $\{x \mid x \in R\}$

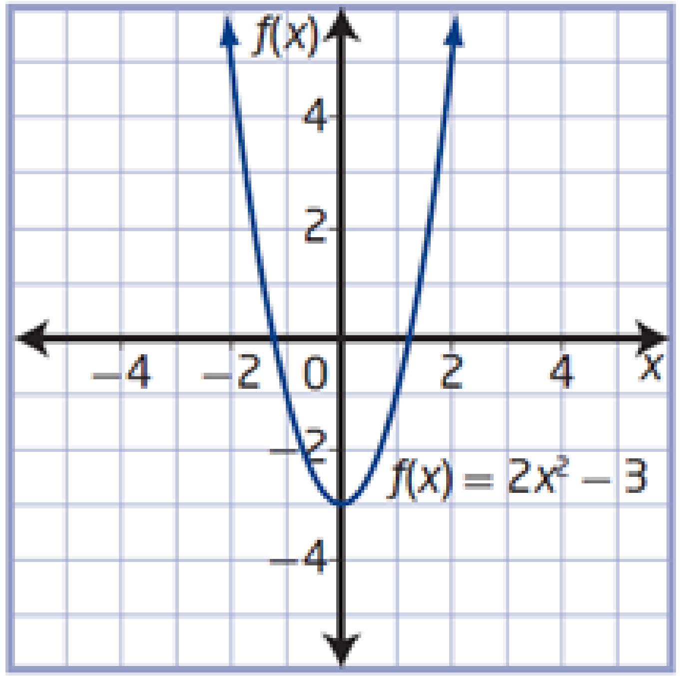
Range: $\{y \mid y \in R\}$

Number of *x*-intercepts: 1

Degree 2: Quadratic Function

Even degree

Number of x-intercepts: 0, 1, or 2



Example: $f(x) = 2x^2 - 3$

End behaviour: curve extends up into quadrant II and up into quadrant I

Domain: $\{x \mid x \in R\}$

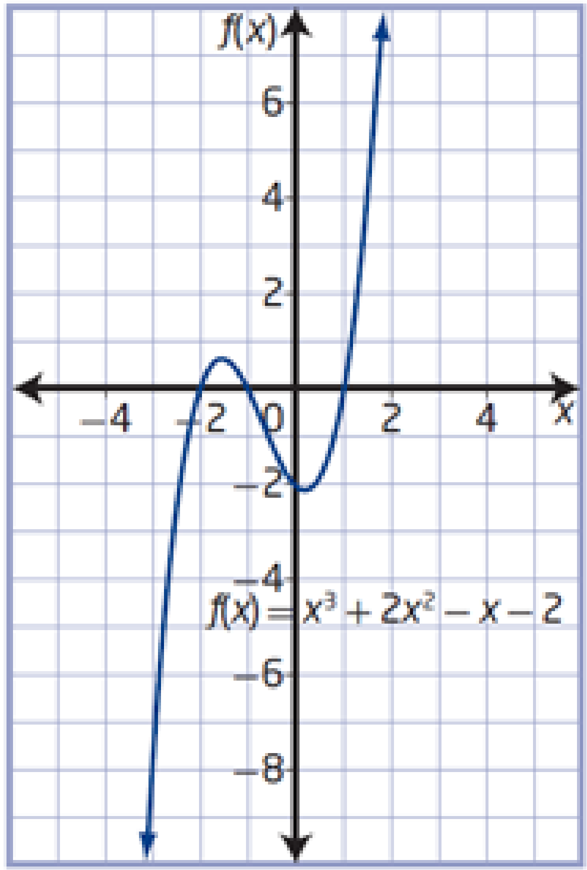
Range: $\{y \mid y \geq -2, y \in R\}$

Number of x-intercepts: 2

Degree 3: Cubic Function

Odd degree

Number of x-intercepts: 1, 2, or 3



Example:

$f(x) = x^3 + 2x^2 - x - 2$

End behaviour: curve extends down into quadrant III and up into quadrant I

Domain: $\{x \mid x \in R\}$

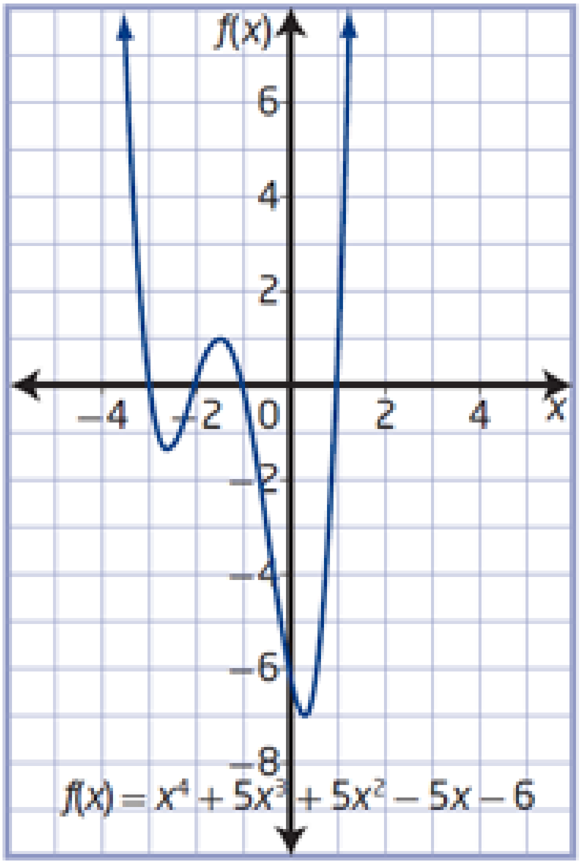
Range: $\{y \mid y \in R\}$

Number of x-intercepts: 3

Degree 4: Quartic Function

Even degree

Number of x-intercepts: 0, 1, 2, 3, or 4



Example:

$f(x) = x^4 + 5x^3 + 5x^2 - 5x - 6$

End behaviour: curve extends up into quadrant II and up into quadrant I

Domain: $\{x \mid x \in \mathrm{R}\}$

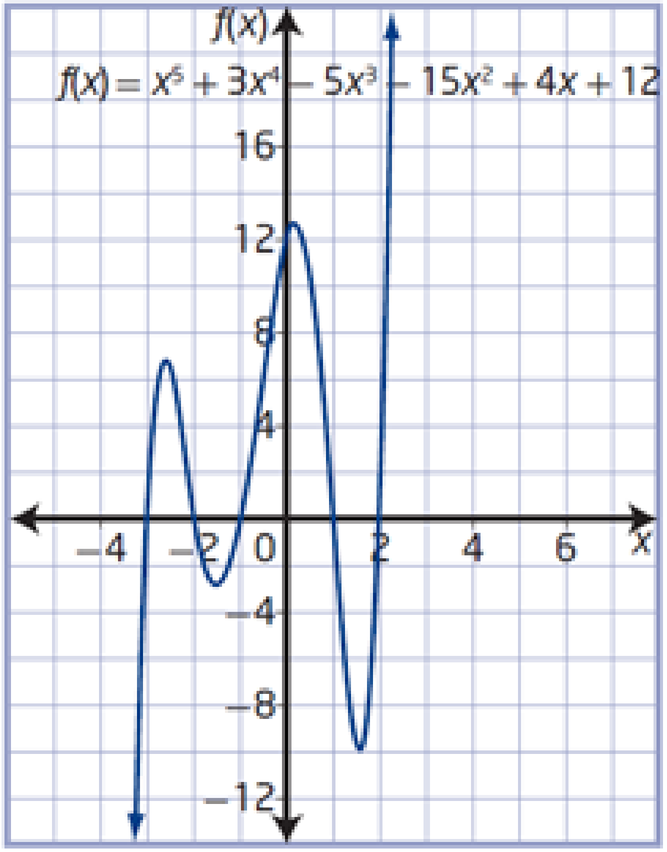
Range: $\{y \mid y \geq -6.91, y \in \mathrm{R}\}$

Number of x-intercepts: 4

Degree 5: Quintic Function

Odd degree

Number of x-intercepts: 1, 2, 3, 4, or 5



Example:

$f(x) = x^5 + 3x^4 - 5x^3 - 15x^2 + 4x + 12$

End behaviour: curve extends down into quadrant III and up into quadrant I

Domain: $\{x \mid x \in R\}$

Range: $\{y \mid y \in R\}$

Number of x-intercepts: 5