1.2 Reflections and Stretches

A **stretch**, unlike a translation or reflection, changes the shape of the graph. However like translations, stretches do not change the orientation of the graph.

$$y = f(x) \longrightarrow y = af(x)$$

$$(x, y) \xrightarrow[\text{maps to}]{} (x, ay)$$

$$\left.\begin{array}{l} y = f(x) \longrightarrow y = af(x) \\ (x, y) \longrightarrow (x, ay) \end{array}\right\} \text{vertical stretch}$$

$a > 1 \Rightarrow$ taller (when the x-values are the same)

$0 < a < 1 \Rightarrow$ shorter (when the x-values are the same)

Examples: Vertical Stretches $\boxed{af(x)}$

① $f(x) \longrightarrow 4f(x)$ $a=4$

$(x,y) \longrightarrow (x,4y)$

$(2,1) \longrightarrow (2,4)$ taller than $f(x)$ at $x=2$

② $f(x) \longrightarrow \frac{1}{5}f(x)$ $a=\frac{1}{5}$

$(x,y) \longrightarrow (x,\frac{1}{5}y)$

$(6,20) \longrightarrow (6,4)$ shorter than $f(x)$ at $x=6$

You Try:

① $f(x) \longrightarrow \frac{1}{4}f(x)$

$(x,y) \longrightarrow (x, \frac{1}{4}y)$

$(3,12) \longrightarrow (3,3)$

② $f(x) \longrightarrow 6f(x)$

$(x,y) \longrightarrow (x, 6y)$

$(3,5) \longrightarrow (3,30)$

$$f(x) \qquad\qquad f(bx)$$

$$(x, y) \xrightarrow[\text{maps to}]{} \left(\frac{1}{b}x, y\right)$$

$\Big\}$ horizontal stretch

if $b > 1$ → narrower (less spread out)

when the y-values are the same, the x-values are closer to the y-axis.

if $0 < b < 1$ → wider (more spread out)

when the y-values are the same, the x-values are further away from the y-axis.

Examples: Horizontal Stretches $\boxed{f(bx)}$

① $f(x) \longrightarrow f(\frac{1}{4}x)$ $b=\frac{1}{4}$

$(x,y) \longrightarrow (4x,y)$

$(2,7) \longrightarrow (8,7)$ further from the y-axis (more spread out)

② $f(x) \longrightarrow f(3x)$ $b=3$

$(x,y) \longrightarrow (\frac{1}{3}x,y)$

$(6,-1) \longrightarrow (2,-1)$ closer to the y-axis (less spread out)

You Try:

(1) $f(x) \longrightarrow f\left(\frac{1}{3}x\right)$ $b = \frac{1}{3}$

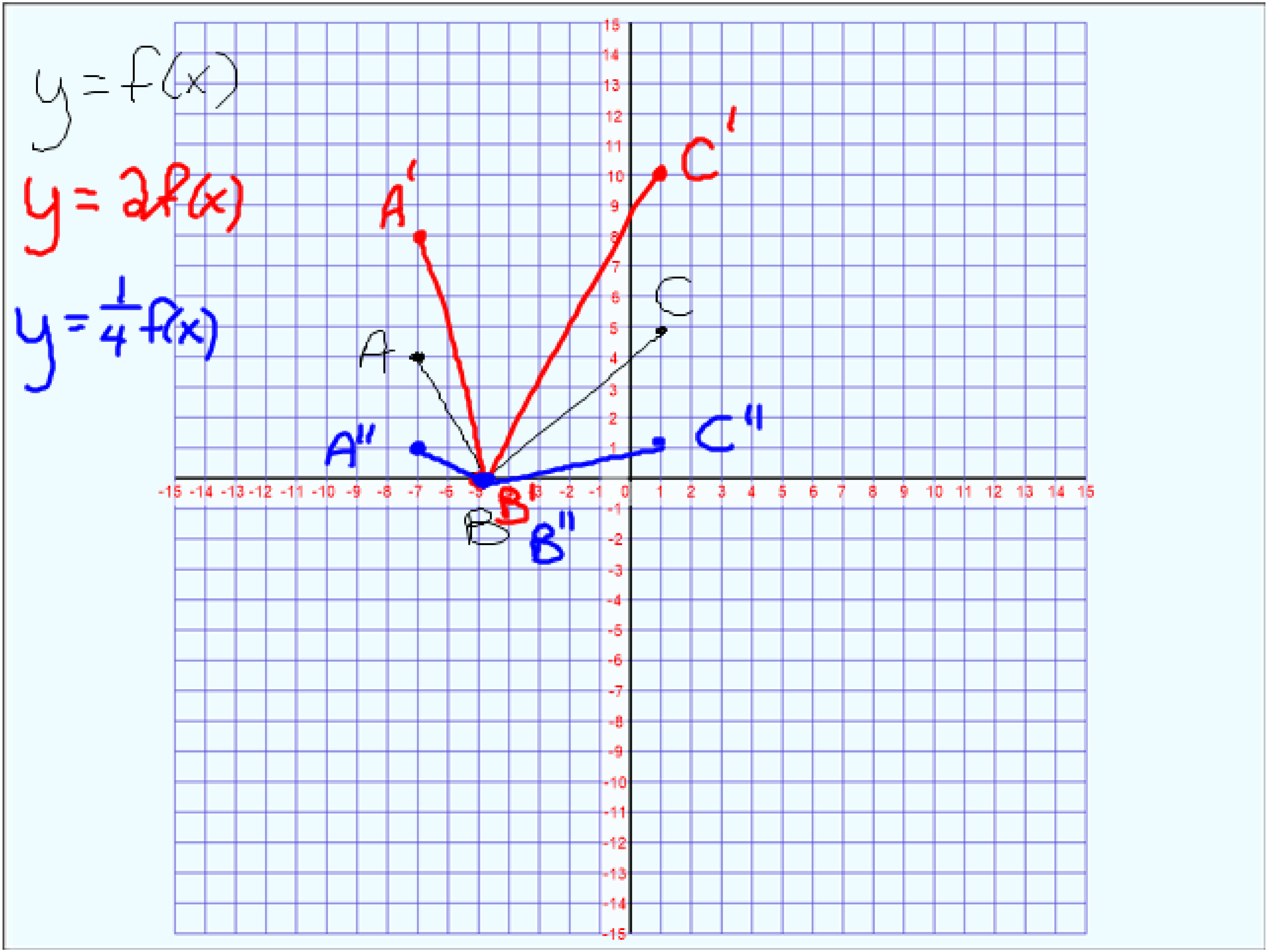
$(x, y) \longrightarrow (3x, y)$

$(1, 7) \longrightarrow (3, 7)$

(2) $f(x) \longrightarrow f(6x)$ $b = 6$

$(x, y) \longrightarrow \left(\frac{1}{6}x, y\right)$

$(18, 1) \longrightarrow (3, 1)$

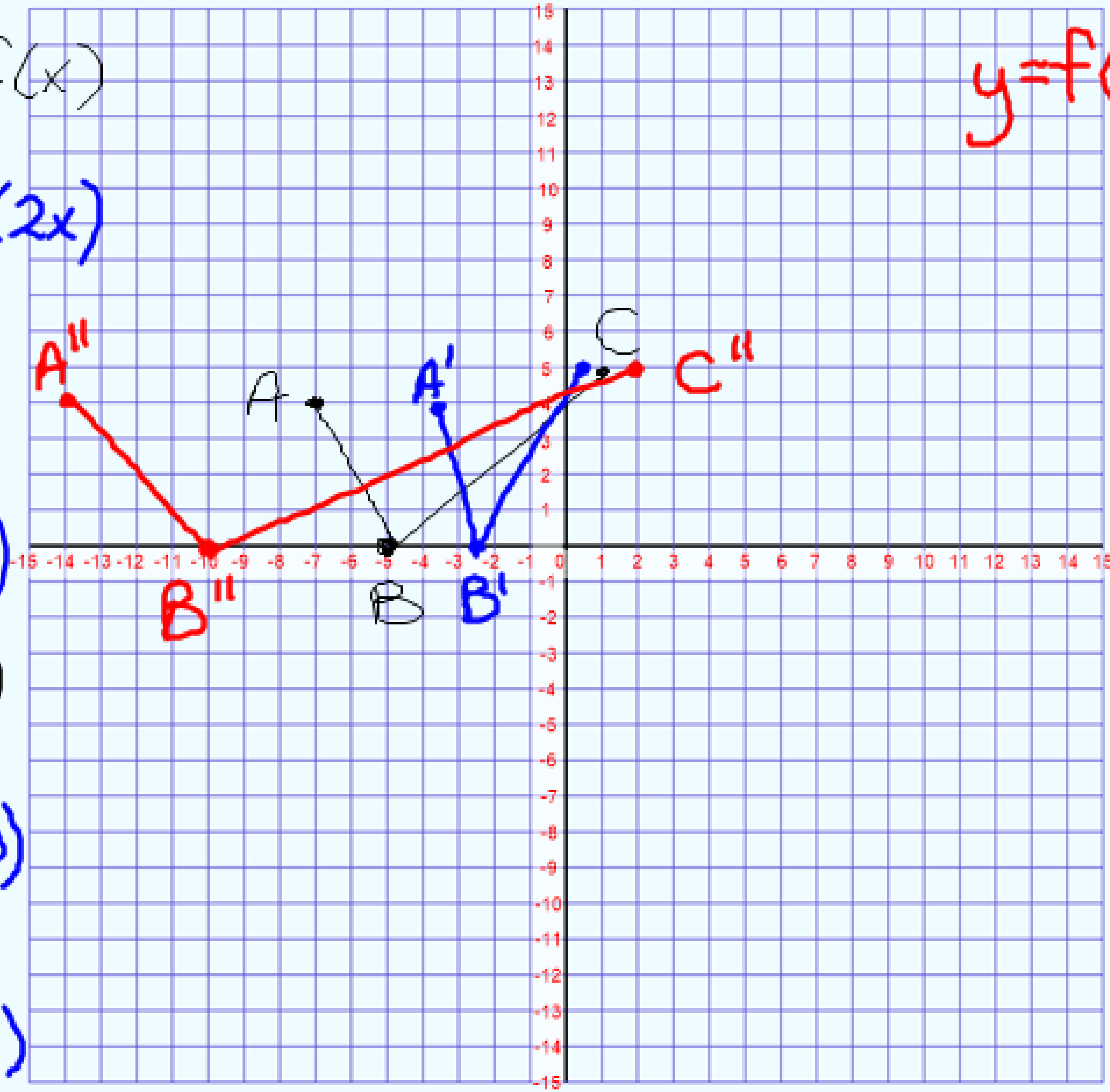


y = f(x)
y = 2f(x)
y = 1/4 f(x)
A'
C'
A
C
A''
C''
B
B'
B''

$y = f(x)$

$y = f(2x)$

$y = f(\frac{1}{2}x)$



$(-7,4)$

$\downarrow$

$(-3.5,4)$

$(-5,0)$

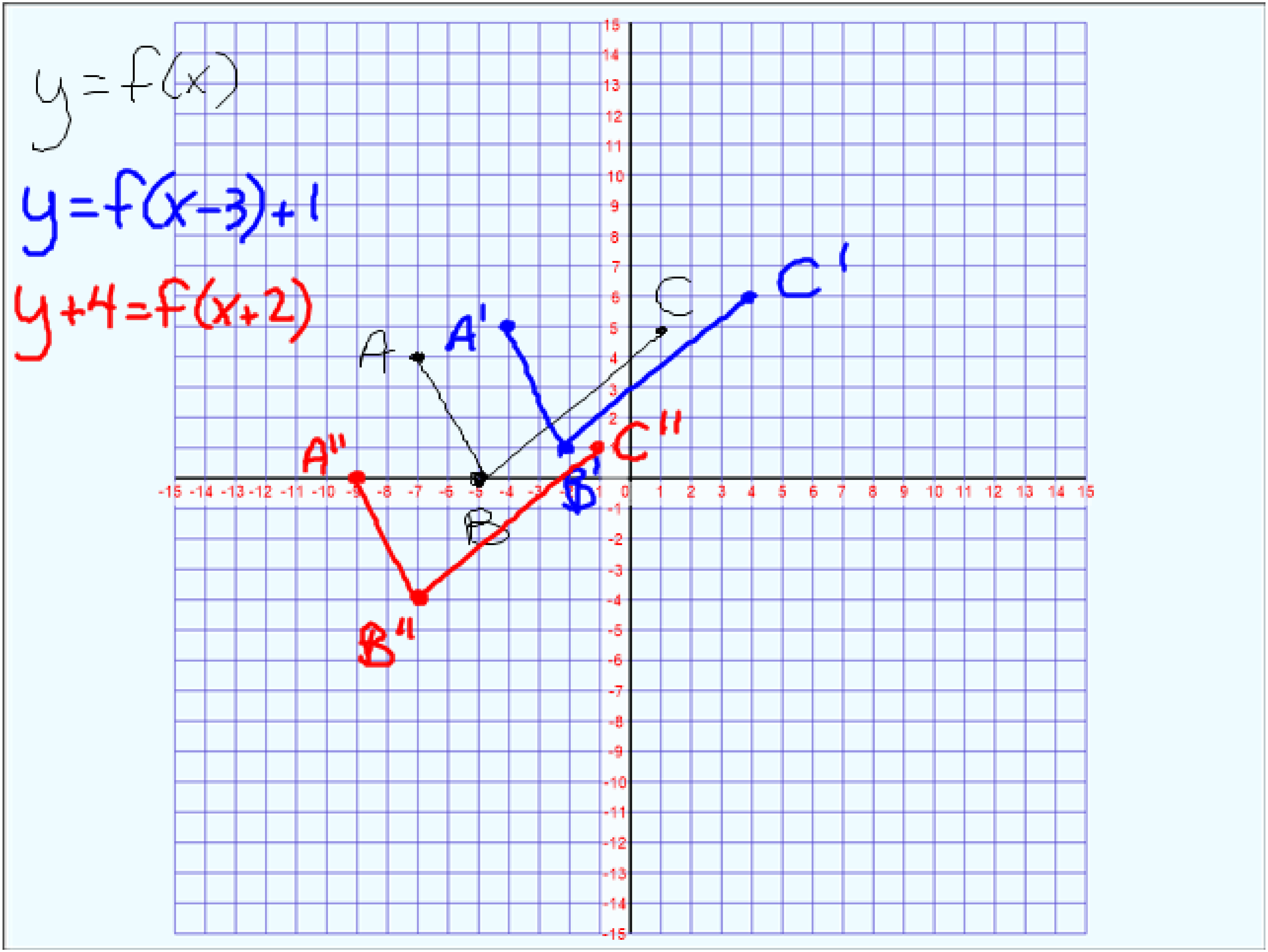
$\downarrow$

$(-2.5,0)$

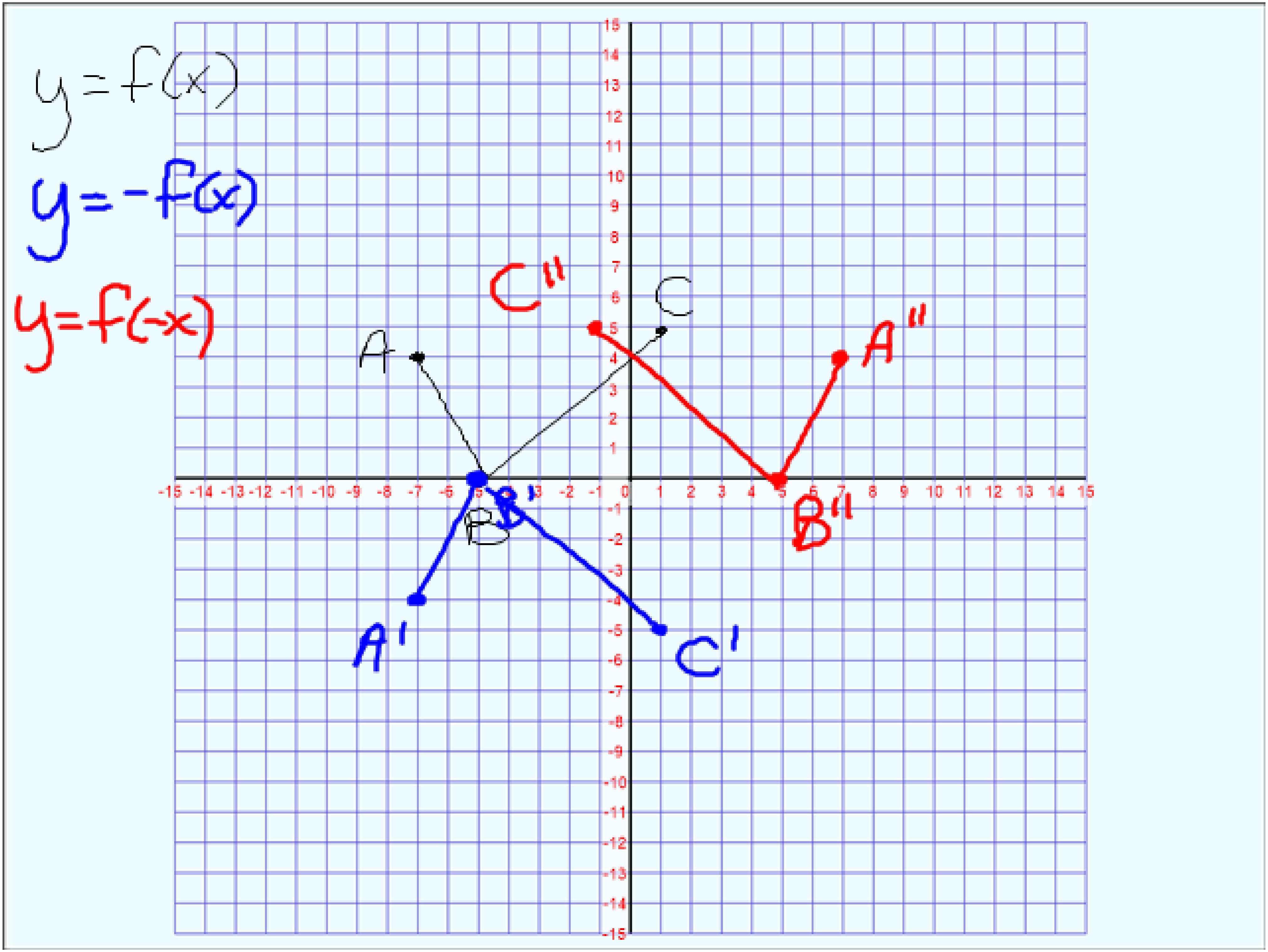
$(1,5)$

$\downarrow$

$(0.5,5)$



y = f(x)
y=f(x-3)+1
y+4=f(x+2)
A
A'
A''
B
B'
B''
C
C'
C''



y = f(x)
y = -f(x)
y = f(-x)
A
B
C
A'
B'
C'
A''
B''
C''

Domain and Range

How do transformations affect the domain and range of a function?

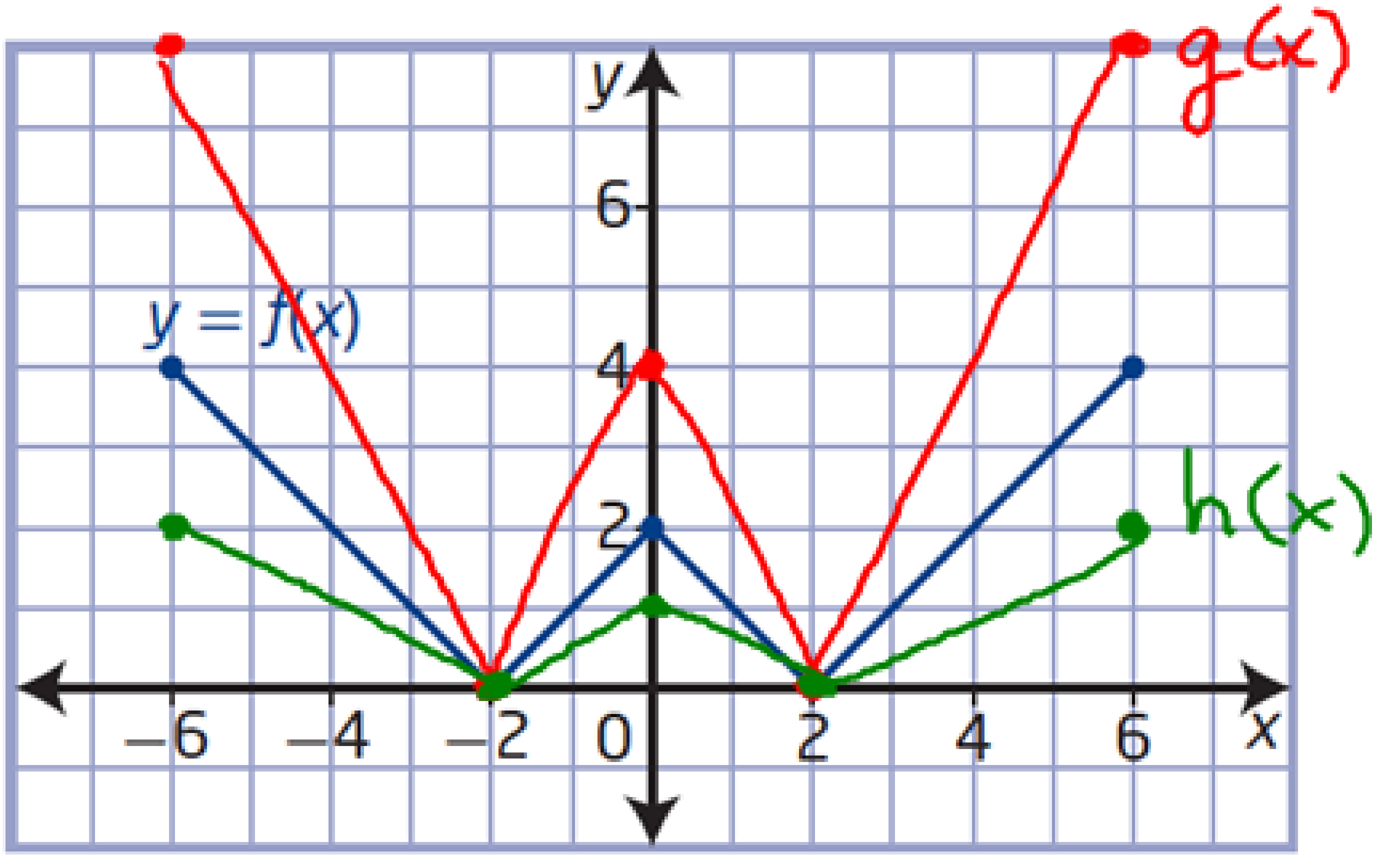
Let's re-examine two of the graphs from yesterday.

Vertical Stretches: $y = a f(x)$

Given $y = f(x)$, graph:

a) $g(x) = 2 f(x)$ → $a = 2$

b) $h(x) = \frac{1}{2} f(x)$ → $a = \frac{1}{2}$



Domain (f) : $\{-6 \le x \le 6 \,,\; x \in \mathbb{R}\}$

Domain (g) : $\{-6 \le x \le 6, x \in \mathbb{R}\}$ no changes

Domain (h) : $\{-6 \le x \le 6, x \in \mathbb{R}\}$

Range (f) : $\{0 \le y \le 4, y \in \mathbb{R}\}$

multiply by $a = 2$

Range (g) : $\{0 \le y \le 8, y \in \mathbb{R}\}$

Range (h) : $\{0 \le y \le 2, y \in \mathbb{R}\}$

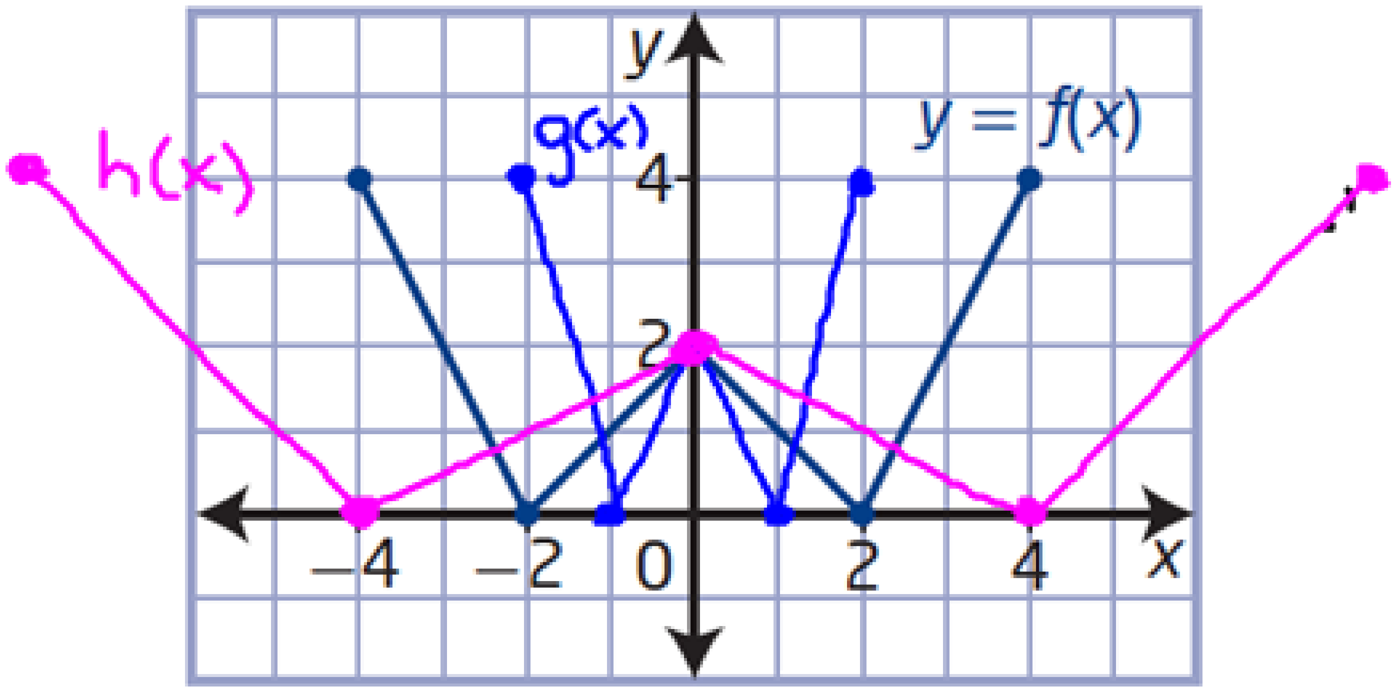
multiply by $a = \frac{1}{2}$

Horizontal Stretch: $y=f(bx)$

Given $y = f(x)$, graph :

a) $g(x) = f(2x)$ → $b=2$

b) $h(x) = f(\frac{1}{2}x)$ → $b=\frac{1}{2}$



Domain (f) : $\{-4 \le x \le 4, x \in \mathbb{R}\}$

Domain (g) : $\{-2 \le x \le 2, x \in \mathbb{R}\}$

Domain (h) : $\{-8 \le x \le 8, x \in \mathbb{R}\}$

multiply by $\frac{1}{b} = \frac{1}{2}$

multiply by $\frac{1}{b} = 2$

Range (f) : $\{0 \le y \le 4, y \in \mathbb{R}\}$

Range (g) : $\{0 \le y \le 4, y \in \mathbb{R}\}$

Range (h) : $\{0 \le y \le 4, y \in \mathbb{R}\}$

no change